Enhanced Multicarrier Techniques for Professional Ad-Hoc and Cell-Based Communications (EMPhAtiC)

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FB-MC and Enhanced OFDM Schemes

Abstract:
The EMPhAtiC project addresses the challenging new spectrum use scenarios encountered in opportunistic dynamic spectrum access and cognitive radio. The focus is on the application in broadband Professional Mobile Radio (PMR) system development for a coexistence scenario with existing narrowband systems. One of the most crucial steps in this development is the selection of suitable waveform(s) with sufficient spectral containment and flexibility for effectively utilizing the available fragmented spectrum. This document investigates advanced multicarrier waveforms in this context, considering different variants of filter bank multicarrier (FB-MC) and single carrier (FB-SC) waveforms as well as methods for enhancing sidelobe suppression in CP-OFDM. Also spectrally efficient non-uniform multicarrier waveforms, with configurable subchannel bandwidths, are investigated as a way to adapt different users’ waveforms to their local radio scene and communication requirements.
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1. Introduction

1.1 Motivation

Multicarrier modulation has most of the key elements needed in the challenging new spectrum use scenarios, like opportunistic dynamic spectrum access, cognitive radio, and heterogeneous wireless system coexistence. Characteristic to these situations is the need to adjust the spectral characteristics of the transmitted signal, notably bandwidth and center frequency, to the available unused slots of radio spectrum. To support high data rates, it is often desirable to combine multiple non-contiguous spectrum slots in the transmission. In multicarrier systems, this can be achieved by activating only those subcarriers that are within the available frequency slots.

Orthogonal frequency-division multiplexing (OFDM) using the time-guard interval known as cyclic-prefix (CP) is the most important multicarrier technique and it is extensively utilized in modern broadband radio access systems. This is due to the simple and robust way of doing channel equalization, high flexibility and efficiency in allocating spectral resources to different users, as well as simplicity of combining multiantenna schemes with the core functionality [1]. However, OFDM has one major limitation in the mentioned coexistence scenarios: limitations in spectral containment, which leads to spectral leakage of the transmitted signal and high sensitivity to interferences from asynchronous spectral components, e.g., in fragmented spectrum use. The traditional channel filtering approach is commonly used for isolating the used frequency band from adjacent channels. However, in case of dynamic and/or fragmented spectrum use, the needed channel filtering becomes difficult to implement and may extend the combined delay spread of channel and filtering beyond the CP length. Various techniques to reduce the level of side lobes in the OFDM spectrum are available in the literature [2] [3] [4] [5] [6]. However, each of them has its limitations in terms of overheads in the transmission capacity and/or implementation complexity versus efficiency of side lobe suppression.

Besides the out-of-band spectral leakage in ideal situation, very important is to assure spectral compatibility in realistic situations where the non-linear transmit power amplification is involved. In that case the number of subcarriers, i.e. sub-channels may play an important role, and the CP-OFDM has significant disadvantage when it comes to preserving reasonably good spectral efficiency. For these reasons consideration of alternative multi-carrier modulation formats for the emerging wireless applications becomes necessary.

1.2 Objectives

An alternative scheme for the considered scenarios is offered by the filter bank based methods of waveform processing and channelization filtering [7]. Actually, it is possible to combine both functions in filter bank based implementations:

- The waveforms generated for transmission are spectrally well-contained and no other measures are needed to clean the unused portions of the spectrum allocated for dynamic/fragmented use.
- The filter bank processing on the receiver side is able to suppress the interferences in the unused parts of the allocated spectrum.
As mentioned above, there are limitations in the reachable levels of attenuation, mostly determined by the analog RF imperfections, notably power amplifier nonlinearity on the transmitter side and nonlinearity of the active stages of the receiver chain, as well as I/Q imbalance effects on both sides. These aspects will be addressed in future deliverables of EMPhAtiC.

The focus of this Deliverable is to evaluate the feasibility of different multicarrier waveforms for the broadband PMR scenario and to select and derive the most promising candidates for further development, evaluation, and demonstrator implementation. The preliminary evaluations have included enhanced OFDM schemes as well as different filter bank based (uniform and non-uniform) multicarrier (FB-MC) and so-called single carrier (FB-SC) schemes.

One of the core ideas is to investigate the feasibility of a special implementation scheme for multirate filter banks which is based on fast-convolution (FC) processing. The basic idea of fast convolution is that a high-order filter can be implemented effectively through multiplication in frequency domain, after taking DFT’s of the input sequence and the filter impulse response. Eventually, the time domain output is obtained by IDFT. Commonly, efficient implementation techniques, like FFT/IFFT, are used for the transforms, and overlap-save processing is adopted for processing long sequences. The application of FC to multirate filters have been presented in [8], and FC implementations of channelization filters has been considered in [9, 10, 11]. The idea of FC-implementation of nearly perfect-reconstruction filter bank systems has been introduced in [12] and detailed analysis and FC-FB optimization methods will be reported in [13].

In Section 2., after an overview and classification of multi-carrier schemes, first are introduced and evaluated various forms of the enhanced (CP-)OFDM schemes, time-frequency dual of the FBMC/OQAM time-limited orthogonal (TLO) scheme is revisited, followed by the non-uniform TLO and FLO (frequency-limited orthogonal) multi-carrier arrangements. This section ends with the variant of the filtered multi-tone (FMT) format, which is based on time-domain windowing of CP-OFDM signals over multiple symbol intervals. Orthogonality conditions for the non-uniform subchannel stacking are derived in Section 3, while the filter-bank implementation framework relevant for the considered multi-carrier schemes is provided in Section 4. This section includes detailed discussion of FC-FB analysis and optimization methods, as well as characterization of the resulting designs in terms of spectral containment and implementation complexity. The second part of this section discusses implementation aspects of polyphase filter banks and FC-FBs. Section 5 covers various aspects of prototype pulse optimization, concerning the out-of-band radiation and channel frequency selectivity.
2. Overview of Multi-Carrier schemes

The classification of the orthogonally frequency-division multiplexed multi-carrier schemes can be based on the features of the presence/absence of the time-frequency overlapping of the signalling waveforms. Three main classes can then be derived: (i) spectrally overlapped, time non-overlapped, (ii) time-overlapped, spectrally non-overlapped, and (iii) those overlapped in both domains. While the first two cases assume spectral inefficiency due to the reliance on guard intervals in time (CP-OFDM) or guard bands in frequency (FMT), the third one generally offers the best utilization of the available spectrum bandwidth. The spectral shaping associated with the time overlapped schemes introduces latency in the transceiver chain, and has to be considered as an important factor in comparative evaluations. One possible classification is shown in the following table.

Table 2.1. Classification of multicarrier and single-carrier FDMA waveforms

<table>
<thead>
<tr>
<th>1. MULTICARRIER MODULATION METHODS</th>
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<tr>
<td><strong>DFT-MC</strong></td>
<td>DFT-based MultiCarrier Modulation</td>
</tr>
<tr>
<td>CP-OFDM</td>
<td>Orthogonal Frequency Division Multiplexing with Cyclic Prefix (the common OFDM scheme)</td>
</tr>
<tr>
<td>Enhanced OFDM</td>
<td>CP-OFDM with additional processing to reduce the level of spectral sidelobes</td>
</tr>
<tr>
<td>FB-MC</td>
<td>Filterbank based MultiCarrier Modulation</td>
</tr>
<tr>
<td>FBMC/OQAM</td>
<td>FB-MC with QAM subcarrier modulation</td>
</tr>
<tr>
<td>TLO</td>
<td>Time-Limited Orthogonal multicarrier modulation: FBMC/OQAM type scheme with time-frequency dual low-complexity prototype filter</td>
</tr>
<tr>
<td>FMT</td>
<td>Filtered MultiTone: non-overlapping subchannels, QAM modulation</td>
</tr>
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<tr>
<th>2. SINGLE-CARRIER FDMA METHODS (SC-FDMA)</th>
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<tbody>
<tr>
<td><strong>DFT-SC</strong></td>
<td>DFT based SC-FDMA</td>
</tr>
<tr>
<td>Basic DFT-SC</td>
<td>The basic scheme known from 3GPP-LTE uplink</td>
</tr>
<tr>
<td>Enhanced DFT-SC</td>
<td>DFT-SC with sidelobe reduction</td>
</tr>
<tr>
<td><strong>FB-SC</strong></td>
<td>Filterbank based SC-FDMA</td>
</tr>
<tr>
<td></td>
<td>FB-spread FBMC/OQAM</td>
</tr>
<tr>
<td></td>
<td>Proposed fast-convolution based flexible SC scheme</td>
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The DFT-MC waveforms include conventional CP-OFDM scheme and its various modifications towards improvement of their sub-channel spectral containment. The second class, FB-MC includes multicarrier formats with essentially Nyquist-shaped subchannel spectra (FBMC/OQAM) or the dual impulse response shapes (TLO), along with the FMT scheme or the maximally spectrally-efficient Partial Response Signalling sub-channel spectra. The single-carrier FDMA methods include the OFDM-based SC-FDMA scheme, the corresponding filter bank based structure [78], as well as FC-FB based scheme for flexible SC waveform generation.

While this classification applies to the traditional uniformly spaced frequency-division multiplex, it may also hold for the corresponding non-uniform subchannel arrangements.
2.1 OFDM Sidelobe Suppression Techniques

The spectrum leakage, in the form of high sidelobes, limits the applicability of OFDM in dynamic fragmented spectrum usage scenarios, since filtering solutions are not practical for cleaning the spectrum in such cases [71]. However, various studies proposing techniques for suppressing the OFDM sidelobes have been presented in the literature. Intuitively, to suppress the OFDM sidelobes, additional processes are required to be included to OFDM implementations. Besides, there may be some losses in the main communication resources (time, frequency, power). Therefore, a tradeoff has to be made carefully in order to achieve sufficient suppression performance with reasonable costs in other respects.

In this chapter, four common techniques for OFDM sidelobe suppression are discussed in detail, showing their basic ideas, mathematical formulations, implementations, and performance. The techniques are evaluated according to their suppression performance, simplicity and ability to be combined with a specific OFDM scheme, possibly including other enhancements, like peak-to-average power ratio (PAPR) control. The sidelobe suppression performances are compared in Section 2.1.5 in a 5 MHz 3GPP LTE scenario.

2.1.1 Time Domain Windowing

The main reason for the high sidelobes of OFDM is the rectangular shape of OFDM symbols in time domain. Unfortunately, this shape has the disadvantage of producing sinc-function in frequency domain\(^1\). Time domain windowing technique intends to reduce OFDM sidelobes by modifying the OFDM symbol shape. This modification aims to add slopes to the OFDM symbol edges, making the transitions between OFDM symbols longer and smoother. This transition is added by multiplying extended OFDM symbols in time domain with an appropriate window that provides the needed smoothness [62]. Time domain windowing must not be confused with common frequency domain windowing; in this chapter windowing always refers to windowing in time domain.

The window cannot be chosen in an arbitrary way; there are some conditions that have to be satisfied. Firstly, the chosen window should have the ability to control the transition length. Secondly, the window must not modify the original data in OFDM symbols; hence extra cycled data symbols have to be appended to the beginning (pre-window interval) and to the end (post-window interval) of the OFDM symbol as depicted in Figure 2-1. In order to minimize the introduced time overhead, two consecutive OFDM symbols are allowed to overlap in the pre-window and post-window intervals, as shown in Figure 2-2. The overlapping period has the length of \(N_w\) samples or \(T_w\) seconds, such that overall OFDM symbol interval becomes \(N_{\text{wnd}} = N + N_{\text{CP}} + N_w\) samples or \(T_{\text{wnd}} = T_u + T_{\text{CP}} + T_w = T_s + T_w\) seconds. Here \(N\) is the IFFT length (total number of subcarriers), \(N_{\text{CP}}\) is the CP duration in samples, and \(T_s = T_u + T_{\text{CP}}\) is the CP-OFDM symbol length in seconds. The windowed OFDM power spectral density (PSD) becomes:

\(^1\) To be precise, the subcarrier spectrum is sinc-shaped in the continuous-time model of OFDM. When implemented using IFFT, the spectrum shape is actually that of the Dirichlet Kernel \(\frac{\sin(N\pi x)}{\sin(x)}\). For simplicity, we use the sinc model in the discussions of this section.
Here $x_{k,c}$ is the complex data sequence in subcarrier $k$, $W(f)$ is the frequency domain representation of window $w(t)$ which has time duration of $T_{\text{wnd}} + T_w$.

Raised Cosine (RC) window (also known as tapered cosine window or Tukey window) is an appropriate shape for time domain windowing as it provides a controllable smoothness to the windowed OFDM symbol without changing data symbols or CPs in time domain. The RC window is expressed according to the following formula,

$$W_{\text{RC}}(t) = \begin{cases} \frac{1}{2} + \frac{1}{2} \cos \left( \frac{\pi \alpha t}{T_{\text{wnd}}} \right) & \text{if } 0 \leq t < \alpha T_{\text{wnd}} \\ 1 & \text{if } \alpha T_{\text{wnd}} \leq t < T_{\text{wnd}} \\ \frac{1}{2} + \frac{1}{2} \cos \left( \frac{\pi (t-T_{\text{wnd}})}{\alpha T_{\text{wnd}}} \right) & \text{if } T_{\text{wnd}} \leq t < (1+\alpha)T_{\text{wnd}} \end{cases}$$

(2.2)

where $\alpha$ denotes the roll-off factor that controls the length of the window interval with $T_w = \alpha T_{\text{wnd}}$ [62]. The frequency domain representation of the RC window can be expressed in the following way:

$$W_{\text{RC}}(f) = T_{\text{wnd}} \text{sinc}(fT_{\text{wnd}}) \left[ \frac{\cos(\pi \alpha T_{\text{wnd}} f)}{1 - 4\pi^2 T_{\text{wnd}}^2 f^2} \right]$$

(2.3)

The RC window in frequency domain is a sinc function multiplied by a window-dependent factor which reduces the sinc sidelobes. Figure 2-3 illustrates the reduction in the sidelobes with increasing roll-off factor. In the sinc case, the first sidelobe is at the level of -13.3 dB while with roll-off value of 0.5 it is at -17.5 dB. For the second sidelobe, the corresponding values are -17.8 dB and -32.7 dB, respectively. However, with the roll-off of 0.5, $T_w = T_{\text{wnd}}/2$, which means that half of the symbol period is reserved for sidelobe control. Hence, the trade-off between the time overhead and sidelobe suppression performance has to be carefully considered.
Figure 2-3: Sinc and RC function behavior in frequency domain.

Figure 2-4: Windowed CP-OFDM implementation model.

Figure 2-4 depicts the implementation model for windowed CP-OFDM. The CP operation is modified in such a way that it appends cycled data in the window interval. Then an additional windowing block is added for multiplying each data symbol in the window interval with the RC coefficients of Eq. (2.2) and the overlapping of the pre- and post-windows of successive blocks is performed.

The added computational complexity can be defined as follows:

\[ C = 4N_w \]
\[ A = 2N_w. \]

(2.4)

Here \( C \) is the number of real multiplication operations per OFDM symbol, and \( A \) is the number of real addition (or subtraction) operations per OFDM symbol. The resulting computational complexity shows a linear increase which is proportional to the window interval length \( N_w \). Therefore, the added computational complexity of time domain windowing is a relatively low compared to other sidelobe suppression techniques considered in this chapter. However, time-domain windowing induces loss in spectrum...
efficiency due to extended symbol period and loss in power efficiency due to the energy transmitted during the transition intervals.

In general, RC time domain windowing results in significant suppression performance, especially at sidelobes located far away from the OFDM edges. In fact, the suppression performance improves as the sidelobe location is getting farther away from the active OFDM subcarriers. Nevertheless, the suppression performance of time domain windowing on sidelobes close to the active subcarriers is considerably low.

2.1.1.1 Edge Windowing

Edge windowing is an enhanced variant of time domain windowing where different windowing lengths are applied on different groups of subcarriers. Basically, subcarriers closer to a band edge leak power to sidelobes more than the inner subcarriers. Therefore, edge windowing technique divides subcarriers into more than one group, usually two. Then it applies different windows with different lengths to each group. Longer window is applied on the edge group and shorter window is applied on the inner subcarriers [67]. The RC window lengths are denoted as $T_{\text{ed}}$ and $T_{\text{in}}$, respectively. In Figure 2-5, the OFDM symbol shape has different CP lengths for each group. The CP length of the edge group is denoted as $T_{\text{ed}}^\text{CP}$ and the CP time of inner group is denoted as $T_{\text{in}}^\text{CP}$. The total OFDM symbol extension is the same for both groups, i.e., $T_{\text{ed}}^\text{ed} + T_{\text{ed}}^\text{CP} = T_{\text{in}}^\text{in} + T_{\text{in}}^\text{CP}$. Then the effective CP of the inner group is longer than the effective CP of the edge group. This provides more channel delay spread immunity to the inner subcarriers.

Figure 2-5: Shape of edge windowed OFDM symbol in time and frequency domains.

This technique can be implemented simply by processing each group separately, using separate IFFT blocks, CP blocks and windowing blocks for each group. Figure 2-6 shows the basic implementation of edge windowed OFDM model. The orthogonality of the subcarriers is maintained if the multipath delays in the inner and edge subcarriers are shorter than the corresponding cyclic prefixes. The user allocation block is a scheduling process that allocates users experiencing long channel delay spread to inner group with long CP, while the edge group is reserved for users that endure short channel delay spread. By using such scheduling, the OFDM system is able to support users with long delay spread while providing effective sidelobe suppression [69]. The IFFT block length is the same, $N$, for all groups and the unused inputs of the IFFT are set to zero.
Users’ Allocation \{x_0, x_1, x_2, \ldots, x_{N-1}\}
\[ \sum \text{IFFT} \]
\[ CP \]
\[ \text{Short window} \]
\[ + \]
\[ y(t) \]

Figure 2-6: Implementation of edge windowing technique.

The computational complexity of edge windowed OFDM is double compared to the basic widowed OFDM, since separated processing is required for each group. Some savings are possible by using pruning methods in the IFFT implementation, but we don’t consider this possibility in our coarse complexity evaluation. Hence, the added computational complexity depends on the computational complexity of IFFT algorithm that is used in the IFFT block. When using the effective split-radix algorithm [68], the added computational complexity compared to basic CP OFDM, in terms of real multiplications and additions per OFDM symbol, can be formulated as follows:

\[
C = (G-1)[N(\log_2 N - 3) + 4] + 4 \sum_{g=1}^{G} N_w^g
\]

\[
A = (G-1)[3N(\log_2 N - 1) + 5] + 2 \sum_{g=1}^{G} N_w^g
\]

Here \( G \) is the number of subcarriers groups and \( N_w^g \) is the window length of \( g \)th subcarrier group. Hence, the added computational complexity is proportional to number of groups used for edge windowing.

Properly configured edge windowing almost reaches the performance of conventional windowing with the same overall OFDM symbol length (see Section 2.1.5). Similar to conventional windowing, edge windowing performance is enhanced rapidly as the sidelobe location gets farther away from the active subcarriers. Besides, edge windowing with user scheduling can be exploited for simultaneously enhancing the spectral characteristics of the transmitted waveform and for increasing the OFDM system immunity to channel delay spread, especially for users that are located far from transmitter. Furthermore, edge windowing technique can be combined with partial transmit sequence (PTS) method for PAPR mitigation in a computationally effective way. Further details of this combination can be found in [75].

2.1.2 Cancellation Carriers

Basically, the power levels of OFDM sidelobes change instantaneously according to the input symbol values. The Cancellation Carrier (CC) technique aims to reduce the power of sidelobes at predetermined range by adding properly weighted subcarriers to the deactivated band of the OFDM spectrum [63] [65]. Essentially, three major parameters affect the suppression performance. These factors are the number of cancellation carriers, the width of the optimization range, and the scheme for calculating the cancellation carrier.
weight sequences. The number of CCs and the optimization range are defined according to the system requirements and affordable computational complexity. Nevertheless, the weight value calculation is the main issue in the CC technique.

Figure 2-7: Frequency domain representation of OFDM spectrum using CC technique.

Figure 2-7 shows the frequency representation when CC technique is used. The positions of the optimization ranges are chosen to be as close as possible to useful band(s) as possible, where the sidelenes powers are the highest. Furthermore, the positions of CCs have to be close to the optimization range because the impact of CC is stronger on closer sidelenes.

To find out the required weights, firstly the number of CCs, \( M \), has to be defined. Besides, the location of each CC has to be defined as an IFFT input bin \( k \), which is not used as a data subcarrier. An unweighted CC is again modelled by a sinc spectrum:

\[
CC_k = \text{sinc} \left[ T_s \left( f - \frac{k}{T_u} \right) \right]
\]  

Secondly, the optimization range, \( S \), has to be defined which represents a set of sidelenes required to be optimized. The suppression can be applied on all points of the optimization range, but the computational complexity is tremendously increased with the number of optimization points. However, it is sufficient to do the optimization only on the middle between center frequencies of unused subcarriers. Assuming that only subcarrier \( k \) is active, those points are located at

\[
f = \frac{n}{2} + \frac{k}{T}
\]

where \( n \) is an odd integer, and the value at that frequency equals

\[
P_n = \frac{(-1)^{n+3} \cos(n \pi / 2)}{(1+r) \pi n / 2},
\]

where \( r = T_{CP} / T_u \). Consequently, a sidelen peak of OFDM spectrum at certain location \( k_s \) in the optimization range is calculated from the complex subcarrier symbol values in the following way:

\[
P_s = \sum_{k \in K} x_k (-1)^{k-k_s+1} y_k \left(1 + r \right) \left( k - k_s - \frac{1}{2} \right) \pi /
\]
where $A$ is the set of active subcarriers and $y_k = \cos[c\pi(k - k_s - \frac{1}{2})]$ is a periodic function changing according to active subcarrier index and the index of the optimized point. Then, the points of the optimization range are collected in the column vector $P = [P_1, P_2, ..., P_S]^T$.

Thirdly, the sidelobe points of CCs that are located in the optimization range are collected. For a non-weighted CC, they are calculated as follows:

$$C_{s,m} = \frac{(-1)^{k_m-k_s} \cdot 2y_{k,s}}{(1+r)(k_m-k_s-\frac{1}{2})\pi} \tag{2.10}$$

where $k_m$ is the CC index in the IFFT block. $C_{s,m}$ in (2.10) is an element of matrix $C = [C_1, C_2, ..., C_S]$ of $S \times M$ elements. Here columns $C_m = [C_{1,m}, C_{2,m}, ..., C_{S,m}]^T$ represent the points in the optimization range due to one CC. The unknown complex weight vector is defined as $g = [g_1, g_2, ..., g_M]^T$, where $g_m$ is the weight of the $m$th CC. Finally, the following minimization problem is formulated using the generated matrices:

$$\min_g ||P + Cg||^2 \text{ subject to } ||g||^2 \leq a^2 \tag{2.11}$$

where $a$ is the power limitation for cancellation carriers. The problem formulated in equation (2.11) is a least squares problem with quadratic inequality constraint. The solution of this problem can be found in [77].

Figure 2-8 represents an implementation of the OFDM model that utilizes CC technique for sidelobe suppression. The red arrows are the inserted carriers. The weights evaluation block is the only added computational complexity to the OFDM model. However, depending on the specific CC scheme, this block may have relatively high complexity.

![Figure 2-8: Implementation of OFDM model using CC technique for sidelobe suppression.](image)

The computational complexity of CC technique is defined by the number of computations needed to solve the weights. This includes (i) calculating the sidelobe peaks due to data subcarriers using equation (2.9) and (ii) finding the solution of the minimization problem (2.11). The CC sidelobe peak matrix (2.10) is fixed and needs to be calculated only once, or in case of dynamic spectrum use, every time after system reconfiguration.
Figure 2-9 shows the implementation of the weight evaluation block mentioned in Figure 2-8. The CC weight matrix that results from equation (2.10) is used to compute the singular value decomposition (SVD), which is used to transform the minimization problem (2.11) into a simpler form. Therefore, in case of fixed CC configuration and fixed optimization range, the SVD decomposition can be computed once and fed into Lagrangian and CC solver block, which reduces the computational complexity dramatically. Naturally, these calculations have to be repeated after every reconfiguration of the active data subcarriers, CCs and/or optimization range [66].

The computational complexity of CC weight calculation for the example case considered in Section 2.1.5 is estimated in the following. In this case there are four groups of two CCs, and two optimization points (at two closest sidelobe peaks) are used for each pair of CC’s. In other words, there are eight CC’s and eight optimization points in total. The complexity is expressed as the number of real multiplications, C, number of real additions, A, number of real divisions, D, and number of real square root operations, S, each per OFDM symbol. In the following, \( N_u \) is the number of active data subcarriers and \( I \) is the number of Newton iterations needed to solve for the Lagrangian coefficient. In the dynamic case, the CC-matrix and SVD need to be computed and the complexity can be expressed as:

\[
\begin{align*}
C &= 29539 + 51I + 16N_u \\
A &= 38061 + 50I + 16N_u \\
D &= 271 + 17I + 8N_u \\
S &= 72
\end{align*}
\]
The computational complexity in fixed CC case is:

\[ C = 80 + 51I + 16N_u \]
\[ A = 512 + 50I + 16N_u \]
\[ D = 8 + 17I + 8N_u \]
\[ S = 0 \]  

(2.13)

The previous two equations are calculated based on the assumption that \( C \) is a full rank matrix [74]. Formulas at (2.12) and (2.13) show that the difference in computational complexity between the fixed and dynamic case is considerably large. Furthermore, the computational complexity difference increases with increasing number of CCs and increasing optimization range.

Another major aspect that affects the computational complexity is the power limitation of the CCs weights. In minimization problems without constraints, the power of the CC weights may reach high levels, e.g., 10 dB, which increases the PAPR and the power consumption. Accordingly, power limitation provides the guarantee of keeping the power consumption over CCs at an acceptable level. The computational complexity of unlimited CCs case is low because the involvement of the constraints in the minimization problems requires extra computations. The dynamic case of unlimited CC power in the case with eight CCs and eight sidelobes in the optimization range has the following computational complexity:

\[ C = 796 + 16N_u \]
\[ A = 796 + 16N_u \]
\[ D = 142 + 8N_u \]
\[ S = 0 \]  

(2.14)

The fixed case with unlimited CC power in the same CC configuration has the following computational complexity:

\[ C = 64 + 16N_u \]
\[ A = 64 + 16N_u \]
\[ D = 8N_u \]
\[ S = 0 \]  

(2.15)

In conclusion, a significant amount of computations are required to achieve the power limitation. Besides, the CC computational complexity is significantly reduced when the positions of CCs and optimization range is not changing.

The suppression performance of CC technique is expected to be good in the optimization range. But outside the optimization range, the sidelobes power tends to grow towards the sidelobe power of the regular CP-OFDM scheme. However, increasing the optimization range heavily enlarges the required computational complexity. Therefore, a careful choice of the number of needed CCs and the length of optimization range must be made in order to achieve the required suppression with minimum possible complexity. The performance range limitation of CC motivates the usage of CC especially at spectral gaps where the required optimized range is limited [65].
2.1.2.1 **Simplified cancellation carrier scheme**

The sidelobe suppression performance of CC technique is high in the optimization range. But the relatively high computational complexity of CC is still a major obstacle. Hence, there is a need for reducing the complexity of the CC technique in order to achieve an acceptable suppression with the lowest possible computational complexity. We focus on reducing the sidelobe levels in relatively narrow spectral gaps in the non-contiguous OFDM (NC-OFDM) case and assume that just one or two CC’s are inserted at each edge of the gap(s) to optimize the next one or two sidelobes. These cases are referred to as 1CC and 2CC, respectively. Each CC weight (1CC) or pair of weights (2CC) is evaluated independently of the others [76]. Figure 2-10 shows the proposed positions of each CC and the optimization range, where they are optimized separately, for the 2CC case.

![Figure 2-10: Frequency domain representation of OFDM spectrum using simplified CC technique.](image)

In the simplified CC, the implementation of the weight evaluation block is significantly reduced. Next we evaluate the computational complexity for the simplified 2CC method with four pairs of CC’s, each with two optimization points. In the dynamic case with power limitation, the complexity can be expressed as follows:

\[
\begin{align*}
C &= 628 + 60I + 16N_u \\
A &= 780 + 56I + 16N_u \\
D &= 124 + 20 + 8N_u \\
S &= 12 
\end{align*}
\]  

(2.16)

The computational complexity of simplified 2CC in fixed scheme with power limitation is:

\[
\begin{align*}
C &= 32 + 60I + 16N_u \\
A &= 32 + 56I + 16N_u \\
D &= 8 + 20 + 8N_u \\
S &= 0. 
\end{align*}
\]  

(2.17)

The computational complexity of simplified 2CC in dynamic scheme without power limitation is:

\[
\begin{align*}
C &= 100 + 16N_u \\
A &= 100 + 16N_u \\
D &= 40 + 8N_u \\
S &= 0. 
\end{align*}
\]  

(2.18)
The computational complexity of simplified 2CC in fixed scheme without power limitation is:

\begin{align*}
C &= 16 + 16N_u \\
A &= 16 + 16N_u \\
D &= 8N_u \\
S &= 0.
\end{align*}

(2.19)

In comparison to the regular CC technique, the simplified CC scheme offers quite significant reduction in the computational complexity.

As discussed in Section 2.1.5, the suppression performance of the simplified CC is close to suppression provided by the regular CC with the same configuration. This is because the CCs located far from optimization range have weak effect.

### 2.1.3 Polynomial Cancellation Coding

Polynomial Cancellation Coding (PCC) is a technique that is used for reducing OFDM sensitivity to frequency errors and phase errors [70]. The benefits of PCC technique includes the spectral enhancement, i.e., reducing sidelobe powers significantly. PCC divides the useful part of the IFFT block into identical groups. Each group contains \( P \) subcarriers representing one data symbol \( x_k \) in such a way that each subcarrier in the group is multiplied by the coefficients of the following polynomial:

\[ (1 - x)^{P-1} \]

(2.20)

The commonly used number of subcarriers per group is \( P = 2 \), in which case the coefficients of the expression (2.20) are defined as \( a_0 = 1 \) and \( a_1 = -1 \). In general, each group of subcarriers is represented in frequency domain as

\[ Y_{kp+p}(f) = T_u a_p x_k \text{sinc}(fT_u - kP - p) \quad \text{for} \quad p = 0, \ldots, P-1. \]

(2.21)

For \( P = 2 \) the subcarrier group spectrum becomes

\[ Y_{2k}(f) + Y_{2k+1}(f) = x_k \frac{T_u \text{sinc}(fT_u - 2k)}{fT_u - 2k - 1}. \]

(2.22)

The resulting fraction in equation (2.22) reduces the sidelobes of the OFDM spectrum.

![Diagram of PCC-OFDM](image)

Figure 2-11: Implementation of PCC-OFDM for groups of two subcarriers.

The implementation of PCC technique for \( P = 2 \) is depicted in Figure 2-11. There is no additional computational complexity, however the scheme has the major drawback of
reduced spectral efficiency by the factor of two. On the other hand, PCC has also been used as a method for eliminating the need for CP [78].

The resulting suppression performance of PCC is high in subcarriers far from the used subcarriers, similar to time domain windowing case. In fact, in time domain PCC can be represented as OFDM symbols multiplied by a complex window.

### 2.1.4 Subcarrier Weighting (SW)

Each subcarrier in the OFDM spectrum is weighted by the subcarrier data symbol $x_k$ which varies instantly according to sent information. Different combinations of the subcarriers weights yields different spectral shapes at sidelobes. The subcarrier weighting (SW) technique aims to multiply the subcarrier symbols by specific weights in such a way that the sidelobes powers are reduced [64]. The primary task in SW is the subcarrier weight optimization. Two parameters need to be defined for the optimization: (i) range of the subcarrier weights, (ii) optimization range, i.e., the set of sidelobes whose energies are used in the minimization. The importance of the weight range is that that it guarantees certain amount of power for each subcarrier symbol, while fixing the overall transmission power level.

The sidelobe power evaluation and minimization follows the model of Section 2.1.2. Equation (2.10) can be used for determining the PSD peaks of each subcarrier within the optimization range. The peak values are collected in matrix $P$ of size $M \times N_u$, where $M$ is the number of sidelobes in the optimization range. The positive, real subcarriers weights are assigned in the column vector $g = [g_0, g_1, ..., g_{N_u-1}]^T$. Then the following minimization problem can be formulated:

$$
\min_g \| Pg \|^2 \quad \text{subject to} \quad g_{\min} \leq g \leq g_{\max} \quad \text{and} \quad \| X \|^2 = \| \overline{X} \|^2
$$

(2.23)

where $X = [x_0, x_1, ..., x_{N_u-1}]^T$ is the data symbol vector and $\overline{X} = [g_0 x_0, g_1 x_1, ..., g_{N_u-1} x_{N_u-1}]$ is the weighted symbol vector. This is a nonlinear optimization problem with quadratic equality and linear inequality. The solution of this problem can be found in [73].

---

**Figure 2-12: Implementation of SW OFDM**

SW implementation is illustrated in Figure 2-12 where the evaluation block produces the weighted subcarriers that are fed to the IFFT. The computational complexity of SW is high compared to CC technique as the used input matrix has rows of length $N_u$, the number of used subcarriers.
In the SW method, no side information is transmitted about the subcarrier weights. Instead, the idea is that the weight range is small enough so that the random variations in the subcarrier symbol values do not essentially degrade the detection performance. This works well for low-order constellations, BPSK or QPSK, or PSK-type modulations. In those cases, the weighting does not affect the decision regions of the receiver, and just the variations of the subcarrier symbol powers affect the BER performance. However, for high-order QAM constellations the weight range should be small and the sidelobe suppression performance would be quite limited.

In the same way as in CC, the sidelobe suppression performance of SW is high in the optimization range, whilst the performance outside the optimization range is weaker and approaches the CP-OFDM at high distances. Hence, SW usage at narrow gaps is expected to be more efficient than in the guard bands of the overall spectrum.

2.1.5 Sidelobe Suppression Performance in 5 MHz LTE Case Study

A basic reference scheme for EMPhAtiC PMR system studies is the 5 MHz LTE case. It is desirable to reach high level of commonality of the main parameters when developing alternative waveform solutions to the basic LTE scheme. The 5 MHz LTE uses $M = 512$ subcarriers, out of which 300 active subcarriers are used [72]. Actually, the scheme includes 150 active subcarriers on both sides of the unused DC-subcarrier. The 300 active subcarriers are scheduled in resource blocks of 12 subcarriers, i.e., there are 25 resource blocks (RBs).

In time domain, the resource blocks have the length of 0.5 ms, i.e., 3840 samples at the used sampling rate of 7.68 MHz. The RBs are scheduled to different users in 1 ms sub-frames of 2 RBs. Especially in the uplink, the minimum transmission burst length is 1 ms, consisting of 7680 samples. In the basic LTE transmission scheme, 14 OFDM symbols are transmitted during one sub-frame. In the extended cyclic prefix (CP) mode, designed to accommodate longer channel delay spreads, 12 OFDM symbols are transmitted during one sub-frame. Since the broadband PMR system is expected to support large macrocells, the extended CP mode is considered particularly interesting in this application. The duration of the useful part of the OFDM symbol is 512 samples ($T_s = 66.67\,\mu s$). The cyclic prefix length is 36 samples ($4.7\,\mu s$) in the normal mode and 128 samples ($16.7\,\mu s$) in the extended mode.

The granularity of spectrum allocation in fragmented spectrum use is assumed to be at the RB level. We consider an example of non-contiguous spectrum use where two spectral gaps within the 300 subcarriers are reserved for narrowband legacy transmissions, like 25 kHz TETRA channels. One of the gaps is 1 RB wide (12 subcarriers, 180 kHz), the other one 2 RB wide (24 subcarriers, 360 kHz).

All PSD results to compare the sidelobe suppression techniques are obtained for 1000 OFDM symbols. The OFDM symbol extension, including CP and possible window transition, is 128 samples, i.e., quarter of the OFDM symbol, unless otherwise noted. Binary PSK modulation is used for the results presented below. However, it has been verified that the results and conclusions with QPSK modulation are very similar to the BPSK case.

2.1.5.1 Time-domain windowing methods

One choice for the parameterization of the basic windowing method is to transmit 6 symbols per RB and use 128 sample total extension for each OFDM symbol. We consider the
case where the effective CP is the same as in the normal LTE mode, 36 samples. Then the RC slope length can be chosen as $T_s = 92$ samples.

Utilizing the edge windowing model, the cyclic prefix and RC-window are chosen in the same way for the edge subcarriers. For inner subcarriers, we use an RC slope of 32 samples and CP length of 96 samples. The edge subcarrier group width is chosen as 2 RBs (24 subcarriers). Figure 2-13 shows the locations of the inner and edge subcarriers and the two spectral gaps. Overall, there are 120 inner subcarriers and 144 edge subcarriers.

Figure 2-13: 5 MHz non-contiguous LTE case with edge windowing. IN: inner subcarriers, ED: edge subcarriers.
Figure 2-14: Sidelobe suppression results for time-domain windowing. (a) Overall PSD envelope. (b) PSD zoomed into the second gap. (c), (d) Zoomed PSD envelopes at the two gaps.

Figure 2-14 shows first the power spectral density envelope (i.e., the ripples of the frequency responses in stopband regions are not shown) of CP-OFDM and edge windowing. In addition, it shows the contributions of the sidelobes of the inner and edge subcarriers in the gaps. Figure 2-14 shows also the zoomed PSD at the second gap with the frequency response ripples, as well as zoomed PSD envelopes of the two gaps. It is obvious that the contribution of inner subcarriers in gaps is less than the contribution of edge subcarriers. Thus, the resulting total edge windowing performance will follow closely the performance of edge subcarriers in the gaps. The difference between edge windowing and conventional windowing is a fraction of a dB. In the center of the gaps, the interference suppression is about 20 dB or 36 dB higher than that of the basic CP-OFDM for the 1 and 2 RB gaps, respectively. In the wider gap, 50 dB suppression is reached for a bandwidth of about 135 kHz, or 9 subcarrier spacings.
Figure 2-15: 5 MHz non-contiguous LTE case with simplified cancellation carrier techniques with 2 CC’s at each edge of the two gaps.

Figure 2-16: Sidelobe suppression results in terms of PSD envelopes for the cancellation carrier methods with CP lengths of 0 (top) and 128 (bottom).
Figure 2-17: Sidelobe suppression results in terms of PSD envelopes for the cancellation carrier methods with CP lengths of 128. First gap of 1 resource block (top). Second gap of 2 resource blocks (bottom).
2.1.5.2 Cancellation carrier methods

Here we consider only the sidelobe suppression in the two spectral gaps. Figure 2-15 shows the location of the cancellation carriers: one or two CC’s are placed at each gap edge to optimize only the nearest one or two sidelobes to them.

The PSD plots for the CC methods are shown in Figure 2-16 for two different cyclic prefix lengths, 0 and 128 samples. Both conventional and simplified CC schemes are considered. In the 2CC (1CC) case, the conventional cancellation carrier scheme is implemented in the form of eight (four) cancellation carriers jointly optimized to suppress all sidelobes of the two gaps with eight (four) optimization points. In the simplified scheme, each CC or pair of CC’s is optimized independently of the others, using one or two optimization points in 1CC and 2CC schemes, respectively. No power limitation is used here.

With the 2CC scheme and no CP, the cancellation carrier levels are high, as can be seen from the peaks at the gaps edges. It was also observed that for CP length of 36, practically no sidelobe reduction is achieved with any of the considered 1CC and 2CC approaches. What happens is that the closest frequency response notches are moved to the optimization points but the sidelobe peaks are not significantly affected. To handle this situation, much more optimization points would be needed, which increases the algorithm complexity. While even a single CC at each active band edge provides useful sidelobe suppression in the gaps in case of no CP, the sidelobe suppression of 1-CC with practical cyclic prefix lengths is marginal. The 2CC methods provide much better results, especially with the CP length of 128. These observations demonstrate the fact that the analysis and optimization of CC methods have to be done for the specific CP length, and design based on the no-CP model, sometimes presented in the literature, is not practical [65]. Also the choice of the CC scheme and achievable sidelobe suppressions seems to depend heavily on the CP length.

Figure 2-18: PAPR results for CP-OFDM, CC with limited power and CC without power limit.
For these reasons, we focus on the case with 2 CC’s at each band edge with CP length of 128. Figure 2-17 shows the zoomed PSD envelope in the two gaps. Even though no power limitation is used, the CC power levels are no more than 5 dB above the data subcarrier power levels. Comparing with the time-domain windowing results, we can see that edge windowing achieves significantly higher suppression in the middle of the gaps, whereas the cancellation carrier approach provides relatively flat PSD over the gap. In the 1 RB gap with 2CC, the simplified scheme is significantly worse than the conventional method, but in the 2 RB gap the results are similar. In both gaps, about -27 dB sidelobe level is reached.

Figure 2-18 shows the complementary cumulative distribution function (CCDF) of the OFDM symbol power levels for CC schemes and basic CP-OFDM. The plot shows the probability that the peak power of an OFDM symbol exceeds the value on the horizontal axis in reference to the average power. An oversampling factor of 4 and 100000 OFDM symbols are used. It can be seen that the PAPR in both unlimited and limited CC cases is practically identical to the PAPR of CP-OFDM. This is because the increase in the number of occupied subcarriers compared to CP-OFDM is rather small.

2.1.5.3 PCC, SW and combined methods

Figure 2-19 and 2-20 show the PSDs with polynomial cancellation coding and subcarrier weighting methods. In SW, the weights are limited to the range [0.75, 1.25] and the optimization range includes only the two spectral gaps. The CP length of 128 is used here for SW and PCC. It was observed that with these methods, the sidelobe suppression performance doesn’t depend critically on the CP length.

It is also possible to combine different sidelobe control methods. Especially the combination of two low-complexity methods, edge windowing and cancellation carriers, seems to be interesting. The PSD plots for this combination is shown in Figure 2-21 when using 2 CC’s at each edge of the two spectrum gaps. The edge windowing parameters are the same as in Section 2.1.5.1.
Figure 2-19: Sidelobe suppression results for the PCC method with $G = 2$. (a) Overall PSD envelope. (b) First gap of 1 resource block. (c) Second gap of 2 resource blocks.
Figure 2-20: Sidelobe suppression results for the SW method with weights limited to the range $[0.75, 1.25]$ and optimization range covering the two spectrum gaps. (a) Overall PSD envelope. (b) First gap of 1 resource block. (c) Second gap of 2 resource blocks.
Figure 2-21: Sidelobe suppression results for the combined method with edge windowing and 2 CC's at the edges of the spectral gaps. (a) Overall PSD envelope. (b) First gap of 1 resource block. (c) Second gap of 2 resource blocks.
2.1.6 Comparisons and Conclusions

Based on the presented results, we can make the following conclusions regarding sidelobe suppression in the spectral gaps of the considered NC-OFDM scenario:

- Time-domain windowing, PCC and SW methods provide increasing sidelobe attenuation towards the center of the gap.
- CC methods provide the best sidelobe attenuation at the edges of the gaps, next to the additional cancellation carriers which slightly reduce the overall gap width.
- With the considered CC methods, the choice of feasible CC configurations and resulting performance depend heavily on the CP length, while the other methods seem to be relatively independent of the CP length.
- In our example case, PCC and SW provide effective sidelobe suppression in wider range than time-domain windowing. However, both of these methods have significant limitations, either in terms of spectral efficiency (PCC) or computational complexity (SW).
- The combination of edge windowing and CC appears as the most effective method for sidelobe control in the considered NC-OFDM scenario. With 2 CC’s at each band edge, the simplified methods provides similar performance as the conventional method, and also the computational complexity can be considered realistic for practical implementations. The CC power levels are not problematic in this combination, and power limitation is not needed. This leads to further simplification as the power constraint can be ignored in the weight calculation. In the example case, the combined scheme provides below -40 dB sidelobe level in the bandwidths of about 75 kHz in the 1 RB gap and 255 kHz in the 2 RB gap.
- Time-domain windowing methods necessitate the use of relatively long extension to the OFDM symbols (CP and window transition), which significantly reduces the spectral efficiency and power efficiency. The performance of the CC methods depends heavily on the CP length. With shorter CP-lengths, like 36 samples, the performance might be significantly worse that with the studied case with CP length of 128, i.e., quarter of the useful symbol duration. In the PMR application, extended cyclic prefix may actually be needed, especially in large macrocells. The edge windowing method makes it possible to make use of the extended window both for effective sidelobe suppression and for coping with large channel delay spreads.

2.2 Uniform FB-MC

2.2.1 TLO – a time-frequency dual of FB-MC

The difference between standard FBMC, i.e. OFDM/OQAM formats with frequency-domain shaping (which could be termed as the FLO – Frequency-Limited Orthogonal) and the TLO multi-carrier formats [16] consists in the utilization of time-frequency dual impulse responses, as illustrated in Figure 2-22.
It may suffice to only note that the 100% roll-off case has the half-cosine function, that is the Hanning window shape of length $T$, and thus corresponds to the frequency-division multiplexing of minimum shift keying (MSK) signals, while in the case of zero-percent TLO, the referent impulse response has rectangular form of length $T/2$, and motivates the CP-OFDM format extension in the way discussed below.

### 2.3 Non-uniform FB-MC

In order to conciliate the advantages of using wider sub-channels in terms of reduction of PAPR and increase of spectral efficiency in situations when predetermined power spectral density (PSD) masks have to be obeyed, while at the same time being able to separate the adjacent channels by relatively narrow frequency guard-intervals, the need arises for a modification which would enable utilization of sub-channels with differing widths within scattered frequency bands (white-zones), and in particular scarcely available frequency-gaps in the targeted Private Mobile Radio (PMR) and Cognitive Radio applications. Need for use of spectrally overlapped adjacent subchannels of different widths and/or roll-off factors, belonging to two user signals, without mutual interference is another motivation for this.

At the early stage of development, we used the framework of extended OFDM [M. Bellanger, “FBMC physical layer: a primer,” PHYDYAS, Jan 2010 http://www.ictphydyas.org] for computer simulation based evaluation of orthogonality conditions, which is based on frequency-domain implementation of the conventional overlap-and-add filtering method, but we have already integrated the non-uniform FBMC formats into the FC-FB structure. We also experimented with orthogonality for the unsymmetrical roll-off factors (in frequency-domain), to be able to possibly better control the latency inherent in the FBMC format.
2.3.1 Within user(s’) (sub-) bands

Here presented is the derivation of orthogonality conditions for the non-uniform filter-bank stacking as applied within the sub-bands devoted to a particular user signal.

2.3.1.1 Aggregating subchannels for NU-FBMC

A direct extension of the uniform filter-bank towards the non-uniform filter-bank multicarrier (NU-FBMC) configuration is to simply aggregate the subchannels of even-stacked or odd-stacked uniform filter-banks or their combinations [18] [19] [20]. These basic uniform filter-bank arrangements are shown in Figure 2-28. For clarity, the roll-off regions are stylized, actually the RRC type roll-off is used. The subchannels are spaced at $1/T$, corresponding to QAM symbol rate, and the frequency shift between even- and odd-stacked subchannels’ center frequencies is $1/2T$. In the following we construct certain configurations and study the orthogonality conditions in terms of the $Re$ and $j*Im$ signalling instants. In general, the spectral symmetry of the roll-off region is retained from the original uniform filter bank, while the roll-off value is reduced when aggregating the channels.

![Figure 2-28: The even (upper) and odd (lower) uniform filter-banks, M=32.](image)

![Figure 2-29: Example of a NU-FBMC arrangement from odd uniform FBMC.](image)

An example of a NU-FBMC arrangement, based on odd uniform FB, is shown in Figure 2-29. Here sCh1p is obtained by aggregating subchannels 0, 1, . . ., 7, sCh2p is obtained by aggregating subchannels 8, . . ., 11, and sCh3p is obtained by aggregating subchannels 12 and 13. Subchannel sCh4p is directly a subchannel of the basic filter bank. The subchannels on the other side of the spectrum have symmetric stacking arrangement. However, it should be noted that such symmetry is not necessary in the presented NU-FBMC concept. The subchannel weights are obtained by adding up the squares of the weights of the overlapping roll-off regions and taking the square root of the sum. Naturally, if the roll-off shape satisfies the square-root Nyquist conditions, the overlapping roll-off regions add up to a constant passband value, on both sides of which the original roll-off shapes are seen. The resulting
roll-offs of subchannels sCh1p, sCh2p, sCh3p, and sCh4p are 12.5%, 25%, 50%, and 100%, respectively.

For the nonuniform filter-bank arrangement of Figure 2-29, the sequencing of the QAM data samples in the 2x oversampled subchannel sequences can be seen in Figure 2-30. In general, for an odd-stacked filter bank, both I and Q parts of the QAM symbols appear systematically either in the real parts or in the imaginary parts of the data samples, whereas in the even-stacked cases, the sequencing follows the OQAM model. In this example case, all the subchannels follow the oddstacked model (i.e., they are positioned as subchannels of an odd-stacked uniform filter bank), and it has been verified by simulations that their I/Q sequencing follows the odd-stacked model. Figure 2-31 shows another example, which contains a wide DC-subchannel (obtained by merging sCh1p and sCh1m) following the even-stacked model, and the OQAM type data sequencing can be observed for it, while the other subchannels follow the odd sequencing model.

In general, independently of the original filter bank stacking model, the subchannels obtained by aggregating may have even-stacked or odd-stacked characteristics, or neither of those. In the latter case, the subchannel signal should be translated by I/Q mixing to follow either of the two models, before the subchannel sampling rate is reduced to the final symbol-spaced value.
2.3.1.2 Using asymmetrical subchannels for NU-FBMC

The relatively low roll-off factors resulting from the uniform FBMC aggregation are unfavourable for at least two reasons: increased transmission delay and increased PAPR. For these reasons, it would be of interest to keep the NU-FBMC subchannels’ roll-off factors as large as possible, while retaining high flexibility in assigning the subchannel bandwidths. This appears to be possible by directly defining the subchannels frequency-domain samples, as for example illustrated in Figure 2-32. Here, we again start from the odd uniform filter-bank arrangement, define a set on non-uniform subchannels by aggregation, and directly maximally increase the roll-off factors, including also more realistic spectral shapes.

In this arrangement, the maximal roll-off factor is 100%, and minimal one is 50%. The I/Q symbol sequencing is of the even type for the DC subchannel and odd type for the others. A simulation check has confirmed the intrinsic orthogonality of the asymmetric NU-FBMC format, the condition for which is only that the squared samples in the overlapping regions are Nyquist-like symmetrical, no matter what the roll-off factors are on the two sides. Figure 2-33 shows the actual subchannel spectra for NU-FBMC arrangements with both symmetric and asymmetric subchannels.

All the presented and simulated arrangements had only one unused basic subchannel between the positive [0, π] and negative (π, 2π) sides of the spectrum. In practice, a wider gap would be used by increasing the sampling frequency, in order to simplify the analog/digital front-end filter design.
Figure 2-32: Example of an NU-FBMC arrangement with asymmetrical subchannel spectra.

Figure 2-33: Examples of the actual subchannel spectra for NU-FBMC arrangements (a) with symmetrical and (b) with asymmetrical subchannel spectra.
2.4 FMT approach

As mentioned above, using a classical OFDM modulation has some drawbacks, especially when we consider the possibility to implement the coexistence of different systems within the same frequency band.

In fact, due to the OFDM very basic pulse shaping in time (rectangular window), the effect in the frequency domain is a relatively slow decreasing spectrum due to high level side-lobes. Assuming the fact that we are able to cancel specific BB sub-carriers to free frequency channels dedicated to NB, we will encounter the following effects:

- On the one hand, this quite slow decreasing frequency spectrum power on adjacent BB sub-carriers, will add significant noise level in NB channels.
- On the other hand, inserting a high power level NB carrier inside the BB spectrum will result in high ICI (Inter-Carrier Interference) levels, due to lack of BB adjacent sub-carriers resilience among themselves.

Thus, the first step consists in reducing interferences between the BB sub-carriers in order to enable the introduction of a narrowband channel. To carry out that issue, a filtering is applied after the stage of OFDM modulation (IFFT), in order to attenuate the influence of the ICI (the pulse-shaping filtering introduced into this study is similar to TEDS modulation characteristics: see hereafter).

2.4.1 Modified modulation pulse-shaping

The modulated signal, at carrier frequency \( f_c \), shall be given by:

\[
M(t) = \text{Re}\{s(t) * \exp(j(2\pi \cdot f_c \cdot t + \varphi_0))\}
\]

(2.1)

Where:

- \( \varphi_0 \) is an arbitrary phase
- \( s(t) \) is the complex envelope of the modulated signal defined as:

\[
s(t) = \sum_{m=0}^{M-1} \sum_{n=1}^{N} S_m(n) g(t - t_n) \exp(j(\omega_n t + \varphi_n))
\]

(2.2)

Where:

- \( M \) is the number of sub-carriers,
- \( N \) is the number of modulated symbols on each sub-carrier,
- \( T \) is the symbol duration on each sub-carrier,
- \( t_n = nT \) is the symbol time corresponding to modulation symbol \( S_m(n) \),
- \( S_m(n) \) is the modulation symbol at time \( t_n \) on sub-carrier \( m \),
- \( \omega_m = 2\pi f_m \) is sub-carrier angular frequency,
- \( g(t) \) is the ideal symbol waveform, obtained by the inverse Fourier transform of a square root raised cosine spectrum \( G(f) \), defined as follows:
Where $\alpha$ is the roll-off factor, which determines the width of the transmission band at a given symbol rate. For practical implementation, a time limited windowed version of $g(t)$, designed under the constraints given by the specified modulation accuracy and adjacent channel attenuation may be applied. Several use cases will be studied in order to find the best compromise.

For practical implementation, we will use the temporal representation of the filter. Classical OFDM modulation is performed, including IFFT processing, and then, at the very final modulation processing stage, the pulse shaping filter is added. The filter length is chosen to be a multiple of the OFDM symbols (and so a multiple of $N_{\text{fft}} + \text{CP}$ samples = $L^*(N_{\text{fft}} + \text{CP})$). As a consequence, the temporal filter shape is applied to those buffered OFDM symbols, with overlapping and adds processing for successive OFDM symbols.

The rejection performances depend on the shape of the filter used. A filter with a high number of coefficients brings better precisions, rejection and nice spectrum windowing. However, the high number of samples necessary for obtaining such a result at the filtering output generates huge time delays. In that way, the latency added by the user of such a filtered LTE signal has to be taken into account in the design of the pulse-shaping function. The goal will be to limit the additional delay, and thus, we will have to achieve the best performances possible, with the shortest filter in terms of number of coefficients.

To be able to analyse the performances brought by this filtering process, simulations of LTE signal (emission and reception) was performed using the following parameters:

<table>
<thead>
<tr>
<th>Configuration parameters</th>
<th>LTE Standard</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic modulation access mode</td>
<td>OFDMA (downlink mode)</td>
</tr>
<tr>
<td>Frequency carrier</td>
<td>450 MHz</td>
</tr>
</tbody>
</table>
Table 2-3: Parameters used for the simulation of LTE signals

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth</td>
<td>1.4 MHz</td>
</tr>
<tr>
<td>FFT size (Nfft)</td>
<td>128</td>
</tr>
<tr>
<td>Sampling Frequency</td>
<td>1.92 MHz</td>
</tr>
<tr>
<td>Sub-carrier spacing (∆f)</td>
<td>15 kHz</td>
</tr>
<tr>
<td>CP</td>
<td>512/2048*NFFT=1/4</td>
</tr>
<tr>
<td>Symbol duration</td>
<td>160 samples: Nfft+CP</td>
</tr>
<tr>
<td>Number of used sub-carriers (Nused)</td>
<td>72 (6 RBs)</td>
</tr>
<tr>
<td>Pulse shaping filter size</td>
<td>L*(Nfft+CP)</td>
</tr>
<tr>
<td>Roll-off factor</td>
<td>Alpha</td>
</tr>
</tbody>
</table>

Here are typical characteristics of such pulse-shaping filters:

filter length: $L*(Nfft+CP)$, with L=6

with Roll-off (Alpha) = 0.35
2.4.2 Adjacent sub-carrier power Rejection enhancements

2.4.2.1 Basic OFDM sub-carrier removal

The following figure represents the spectrum density of a pure OFDM signal, with one blank sub-carrier, zooming precisely on that removed sub-carrier. It is noticed that after removal of the signal contained on that sub-carrier, the spectrum density at this place is still with a relatively strong amplitude (rejection = 9.8dB only). That is due jointly to the following aspects:

- insertion of the cyclic prefix (which “breaks” OFDM orthogonality at air interface),
- and levels of OFDM pulse-shaping side-lobes of adjacent sub-carriers.

Adjacent sub-carrier rejection = -9.8dB.
2.4.2.2 Sub-carrier removal using filtered OFDM modulation

However, by applying a pulse-shape filtering as described previously, one notice that this rejection level reaches a value of -48dB (Filtered LTE spectrum is shown in red, compared to the previous one, in blue). This confirms the assumption that the OFDM side-lobes have a great influence on adjacent sub-carriers power level.

![Filtered LTE signal with blank sub-carrier](image)

Figure 2-36: Filtered LTE OFDM spectrum with blank sub-carrier. Pulse shaping filter characteristics: L = 8, alpha = 0.35. Adjacent sub-carrier rejection = -48dB.

2.4.2.3 Resource Block (RB) removal using filtered OFDM modulation

The following figure illustrates the rejection gain after the suppression of a succession of sub-carriers (one RB). One can notice that for the 8 central sub-carrier frequencies the improvements in rejection are higher than 50dB.

![Rejection gain after suppression of adjacent sub-carriers](image)

Figure 2-37: The rejection gain after the suppression of adjacent sub-carriers (one RB) Pulse shaping filter characteristics: L = 6, alpha = 0.35.
<table>
<thead>
<tr>
<th>Relative blank sub-carriers position</th>
<th>Rejection achieved for LTE</th>
<th>Rejection achieved for filtered LTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-12.8 dB</td>
<td>-42.1 dB</td>
</tr>
<tr>
<td>2</td>
<td>-16.3 dB</td>
<td>-47.7 dB</td>
</tr>
<tr>
<td>3</td>
<td>-18.6 dB</td>
<td>-50.0 dB</td>
</tr>
<tr>
<td>4</td>
<td>-19.8 dB</td>
<td>-51.3 dB</td>
</tr>
<tr>
<td>5</td>
<td>-20.2 dB</td>
<td>-51.9 dB</td>
</tr>
<tr>
<td>6</td>
<td>-20.4 dB</td>
<td>-52.2 dB</td>
</tr>
</tbody>
</table>

Table 2-4: Rejection performances

The role of the filter is to reduce the residual signal power on the unused sub-carriers to a non significant level (less than -40dB) in order not to degrade the received signal (or emitted signal).

2.4.3 Filter characteristics optimisation on both EVM and rejection criteria's

2.4.3.1 EVM degradations and filter characteristics

EVM measurement makes it possible to quantify performances of the received signal, and especially modulation accuracy. The EVM value represents the quality of the received QAM symbols by calculating the distance between the received point and its ideal position in QAM constellation. In simulation, this parameter was measured after the adequate demodulation of filtered OFDM signal. That is to say that the same filtering process is also applied on receiver side.

Considering the simulation of the transmission chain was performed using ideal conditions: without channel propagation simulation, without AWGN... the calculated EVM represents the degradations brought by the filtering process only.

Here are some demodulation results examples:
The previous figure represents the QAM constellation of the signal received after filtering for various filter configurations.

In the first line in Figure 2-38, one can see the representation of a QAM constellation with EVM about 6% and erroneous received bits. Unlike the result of QAM constellation with EVM<1%, the symbols are spread far enough not to be sure where they should be in theory. On the second line, one notices that for a filter with L=6 size and Roll-off=0.33, the EVM is equal to 4.1%. This, just increasing the size of the filter by 1 OFDM symbol (L=7), one obtains an EVM twice better, about 2%.

### 2.4.3.2 Roll-off factor influence

In the Figure 2-39, presented EVM results obtained after filtered OFDM demodulation for various sizes of filter (L) and Roll-off (α). The points marked by the symbol “*”, represents values with BER>0. As far as BER is concerned, as the simulations are performed using ideal propagation case (no variations in time and frequency; no AWGN), BER>0 results show that the filtering process is destructive: QAM symbol could be retrieved. Of course, such results are discriminatory and will be discarded in the following paragraphs.

![Figure 2-39: EVM results obtained after filtered OFDM demodulation for various sizes of filter (L) and Roll-off (α). Nota: on the curves, points marked with ‘*’ corresponds to filter characteristics which give BER>0.](image)
As a conclusion, depending on roll-off factor chosen, it is possible to achieve good sub-carrier rejection, and quite low EVM results. Of course, it is obvious that increasing the filter length allows us to have far better results, in rejection as well as EVM performances.

As a first conclusion, a filter length of 6 OFDM symbols is really the minimum required. In this case, EVM < 4% and rejection = -39dB for a roll-off factor 0.35.

Increasing the filter length of just one OFDM symbol and we are able to achieve far better results: EVM is nearly divided by 2 and rejection is increased by 6 to 8 dB.

It is clear we could achieve very good results having a filter length of roughly 8 OFDM symbols. In the case of filter length equal 8, 9 or 10, EVM performances are reduced to about 1% of error.

Having such curves of performances, it is then possible to choose the best candidate depending on limits we want to set, and thus, define, L, and α. As we can see, influence of L and α, have sometimes contrary effects on results.

As far as optimization is concerned, it is possible to define two roll-off factors: one for the transmitter filter and another one for the reception filter. In that sense, we will optimize the Tx roll-off factor in order to maximize rejection and optimize Rx roll-off factor to reduce EVM, and of course, try to find the best compromise.

### 2.4.3.3 Asymmetric roll-off factors results

The goal is now to find an optimal point in order to maximize the rejection level of removed sub-carrier and minimize the overall EVM at the same time. Indeed, the curves of EVM and rejection according to roll-off factor of the filter show that the optimal point of each curve does not correspond to the same optimum values. The optimization will lead to the search of a compromise.

Simulations were carried out with the same parameters and with the suppression of only one sub-carrier.

### 2.4.3.4 Filter length = 6 OFDM symbols

![Figure 2-40](image)

Figure 2-40: Representation of the performances curves of rejection and EVM according to the roll-off coefficient alpha used in Tx (transmission) and Rx (reception). L=6
We can easily notice on the EVM curve that the zone for which EVM is lower than 4% locates in the interval \( 0.34 < \alpha_{\text{Tx}} < 0.40, 0.32 < \alpha_{\text{Rx}} < 0.37 \). However, for the rejection, the maximum values are in the interval \( 0.44 < \alpha_{\text{Tx}} < 0.49, 0.26 < \alpha_{\text{Rx}} < 0.34 \). Indeed, it is noted that these two zones do not coincide and it is thus not possible to have the best performances for the two parameters at the same time.

Rejection max: 44.7dB (\( \alpha_{\text{Tx}}: 0.46 \)) Rejection min: 34.9dB

EVM min: 3.8% (\( \alpha_{\text{Tx}}: 0.36, \alpha_{\text{Rx}}: 0.35 \)) EVM max: 5.4%

- EVM at maximum Rejection: 44.7dB, 4.4% (\( \alpha_{\text{Tx}}: 0.46, \alpha_{\text{Rx}}: 0.3 \))
- Rejection at minimum EVM: 39.6dB, 3.8% (\( \alpha_{\text{Tx}}: 0.36, \alpha_{\text{Rx}}: 0.35 \))
- Optimal point: (\( \alpha_{\text{Tx}}: 0.42, \alpha_{\text{Rx}}: 0.32 \)) => Rejection: 42.7dB, EVM: 4.0%

### 2.4.3.5 Filter length = 7 OFDM symbols

For a filter length \( L=7 \), it is clear that there is a common zone for maximizing the rejection and minimizing the EVM. One can easily choose an optimal point for the two criteria.

Rejection max: 46.1dB (\( \alpha_{\text{Tx}}: 0.32 \)) Rejection min: 33.2dB

EVM min: 2.2% (\( \alpha_{\text{Tx}}: 0.28, \alpha_{\text{Rx}}: 0.36 \)) EVM max: 5.6%

- EVM at maximum Rejection: 46.1dB, 2.3% (\( \alpha_{\text{Tx}}: 0.32, \alpha_{\text{Rx}}: 0.33 \))
- Rejection at minimum EVM: 44.2dB, 2.2% (\( \alpha_{\text{Tx}}: 0.28, \alpha_{\text{Rx}}: 0.36 \))
- Optimal point: (\( \alpha_{\text{Tx}}: 0.3, \alpha_{\text{Rx}}: 0.34 \)) => Rejection: 45.4dB, EVM: 2.2%
2.4.3.6 Filter length = 8 OFDM symbols

Rejection max: 47.4dB (AlphaTx: 0.35) Rejection min: 37.8dB
EVM min: 1.1% (AlphaTx: 0.27, AlphaRx: 0.27) EVM max: 6.2%
   • EVM at maximum Rejection: 47.4dB, 1.9% (AlphaTx: 0.35, AlphaRx: 0.24)
   • Rejection at minimum EVM: 42.8dB, 1.1% (AlphaTx: 0.27, AlphaRx: 0.27)
   • Optimal point: (AlphaTx: 0.29, AlphaRx: 0.26) => Rejection: 43.8dB, EVM: 1.2%

2.4.3.7 Filter length = 9 OFDM symbols

Rejection max: 50.1dB (AlphaTx: 0.46) Rejection min: 37.6dB
EVM min: 1.2% (AlphaTx: 0.26, AlphaRx: 0.27) EVM max: 4.9%
   • EVM at maximum Rejection: 50.1dB, 2.6% (AlphaTx: 0.46, AlphaRx: 0.21)
   • Rejection at minimum EVM: 48.1dB, 1.2% (AlphaTx: 0.26, AlphaRx: 0.27)
   • Optimal point: (AlphaTx: 0.26, AlphaRx: 0.27) => Rejection: 48.1dB, EVM: 1.2%
2.4.3.8 Filter length = 10 OFDM symbols

![Graph](image1)

Figure 2-44: Representation of the performances curves of rejection and EVM according to the roll-off coefficient alpha used in Tx (transmission) and Rx (reception). L=10

Rejection max: 49.6dB (AlphaTx: 0.29) Rejection min: 40.0dB
EVM min: 0.9% (AlphaTx: 0.24, AlphaRx: 0.22) EVM max: 4.9%
- EVM at maximum Rejection: 49.6dB, 1.1% (AlphaTx: 0.29, AlphaRx: 0.2)
- Rejection at minimum EVM: 46.3dB, 0.9% (AlphaTx: 0.24, AlphaRx: 0.22)
- Optimal point: (AlphaTx: 0.26, AlphaRx: 0.21) => Rejection: 48.3dB, EVM: 0.95%

2.4.3.9 Filter length = 11 OFDM symbols

![Graph](image2)

Figure 2-45: Representation of the performances curves of rejection and EVM according to the roll-off coefficient alpha used in Tx (transmission) and Rx (reception). L=11

Rejection max: 52.4dB (AlphaTx: 0.36) Rejection min: 44.0 dB
EVM min: 1.0% (AlphaTx: 0.21, AlphaRx: 0.22) EVM max: 4.6%
- EVM at maximum Rejection: 52.4dB, 1.4% (AlphaTx: 0.36, AlphaRx: 0.26)
- Rejection at minimum EVM: 49.6dB, 1.0% (AlphaTx: 0.21, AlphaRx: 0.22)
- Optimal point: (AlphaTx: 0.2, AlphaRx: 0.23) => Rejection: 50.8dB, EVM: 1.0%
2.4.3.10 Filter length = 12 OFDM symbols

Figure 2-46: Representation of the performances curves of rejection and EVM according to the roll-off coefficient alpha used in Tx (transmission) and Rx (reception). L=12

Rejection max: 49.7dB (AlphaTx: 0.24) Rejection min: 43.4dB

EVM min: 1.0% (AlphaTx: 0.21, AlphaRx: 0.18) EVM max: 4.7%

- EVM at maximum Rejection: 49.7dB, 1.1% (AlphaTx: 0.24, AlphaRx: 0.17)
- Rejection at minimum EVM: 48.1dB, 1.0% (AlphaTx: 0.21, AlphaRx: 0.18)
- Optimal point: (AlphaTx: 0.22, AlphaRx: 0.18) => Rejection: 48.7dB, EVM: 1.0%

To conclude, the use of asymmetrical roll-off factors makes it possible to improve both filtering performances to increase rejection, and EVM results to have minimum constellation degradations. Depending on filter length cases, we are able to find optimum configurations (or at least best compromises), and thus define roll-off factors for Tx part and Rx part.

Moreover, some attention has to be given to the filter length since it has impacts on the duration of the corresponding temporal signal, which can lead to some constraints relative to temporal support of the transmitted signal especially for short slot transmission. This aspect has also to be taken into account in the trade-off for selecting an FMT configuration.
3. Orthogonality conditions for non-uniform FB-MC

While in the previous sections the non-uniform orthogonal FB-MC subchannel arrangements were made in an ad-hoc (intuitive) manner, in the following the corresponding orthogonality conditions are derived in a more formal way, starting from the orthogonality conditions of the uniform FB-MC subchannel stacking.

3.1 A pragmatic analytical derivation

3.1.1 Non-uniform FB-MC (FLO – Frequency Limited Orthogonal)

Conditions of intrinsic FLO system orthogonality expressed in time domain provided by [16, Ref. 1] is given by

\[
\text{Re}\left\{ \int_{-\infty}^{\infty} \hat{g}_{n,k}(t) \hat{g}_{m,l}^*(t) dt \right\} = \begin{cases} 
1 & k = l, n = m \\
0 & \text{other}
\end{cases}
\]  

(3.1)

where the first and second indexes are for sub-channel central frequency and \( \frac{T}{2} \) instant, respectively, and \( \hat{g}_{n,k}(t) \) is given by

\[
\hat{g}_{n,k}(t) = f(t - \frac{nT}{2}) \exp\left( j \frac{2\pi k}{T} t + j \varphi_{n,k} \right)
\]  

(3.2)

where:

\[
\varphi_{n,k} = \begin{cases} 
\frac{\pi}{2} & \text{odd} \\
0 & \text{even}
\end{cases}
\]  

(3.3)

\( \tilde{T} \) is QAM symbol period, and \( f(t) \) is impulse response of the square-root Nyquist filter with transfer function \( F(\omega) \).

In [16], for derivation of the uniform FBMC-TLO system it was used the fact that \( \{ \hat{G}_{n,k}(\omega) \} \) the Fourier transform of \( \{ \hat{g}_{n,k}(t) \} \), \( \{ \hat{G}_{n,k}(\omega) \} \) can be used to express the FLO orthogonality conditions in frequency-domain as

\[
\text{Re}\left\{ \int_{-\infty}^{\infty} \hat{G}_{n,k}(\omega) \hat{G}_{m,l}^*(\omega) d\omega \right\} = \begin{cases} 
1 & k = l, n = m \\
0 & \text{other}
\end{cases}
\]  

(3.4)

where:

\[
\hat{G}_{n,k}(\omega) = F(\omega - \frac{2\pi k}{\tilde{T}}) \exp\left(-j \frac{n\tilde{T}}{2} \omega + j \varphi_{n,k} \right)
\]  

(3.5)

By the straightforward designing of the non-uniformly spaced subchannels by aggregation of the uniform FBMC subchannels transfer functions, as shown again as an example in Figure 3-1, it can be shown that the orthogonality conditions of the former can be reduced to conditions of the latter one.
Here, subchannels $i'$ and $j'$ are described by (6) and (7) respectively

\[ \hat{G}_{s,r}(\omega) = \sqrt{\sum_{x=0}^{n} \hat{G}^2_{n-x,k-x}(\omega)} \]  

(3.6)

\[ \hat{G}_{p,q}(\omega) = \sqrt{\sum_{y=0}^{j} \hat{G}^2_{m+y,l+y}(\omega)} \]  

(3.7)

By placing the newly generated transfer functions into (3.4), it follows

\[ \int_{-\infty}^{\omega} \text{Re} \left[ \hat{G}_{s,r}(\omega) \hat{G}^*_{p,q}(\omega) d\omega \right] = \text{Re} \left[ \int_{-\infty}^{\omega} \sqrt{\sum_{x=0}^{n} \hat{G}^2_{n-x,k-x}(\omega)} \left( \sqrt{\sum_{y=0}^{j} \hat{G}^2_{m+y,l+y}(\omega)} \right)^* d\omega \right] \]

Since the complex conjugation (*) can be brought under the square-root, as well as under the squaring operation, and since $(c1 + c2)^* = c1^* + c2^*$, the following expression is produced

\[ \text{Re} \left[ \int_{-\infty}^{\omega} \sqrt{\hat{G}^2_{n-i,k-l}(\omega) + \hat{G}^2_{n-(i-1),k-(l-1)}(\omega) + \cdots + \hat{G}^2_{n-1,k-1}(\omega) + \hat{G}^2_{n,k}(\omega)} \right] \]

Due to spectral confinement to widths 1/T, from all of the products under the root-square operation only $\hat{G}^2_{n,k}(\omega)\hat{G}^2_{m,l}(\omega)$ is different from zero, so that the orthogonality conditions are reduced to those of the uniform FLO FBMC format, i.e.

\[ \text{Re} \left[ \int_{-\infty}^{\omega} \hat{G}_{s,r}(\omega) \hat{G}^*_{p,q}(\omega) d\omega \right] = \text{Re} \left[ \int_{-\infty}^{\omega} \hat{G}_{n,k}(\omega) \hat{G}^*_{m,l}(\omega) d\omega \right] = \begin{cases} 1 & k = l, n = m \\ 0 & \text{other} \end{cases} \]  

(3.8)
3.1.2 Non-uniform TLO (TLO – Time Limited Orthogonal)

Similarly as the frequency domain representations of the subchannels spectra were produced by aggregation of the uniform frequency-domain description, for the non-uniform TLO formats, the aggregation is performed at the corresponding (time-domain) referent impulse responses. Their frequency-domain representations are subsequently produced, and appropriately positioned as in the case of the N-FBMC counterpart. Spectrum of the uniform TLO FBMC format for the even filter banks arrangements is shown in Figure 3-2 (a), and spectrum of the non-uniform TLO FBMC format from the odd filter banks arrangements is shown in Figure 3-2 (b).

![Figure 3-2: NU-FBMC arrangement with (a) symmetrical spectra (b) asymmetrical spectra](image)

As was the case with non-uniform FLO formats, for non-uniform TLO formats the orthogonality conditions can also be shown to reduce to those of the orthogonal case. Since in [16] the orthogonality conditions of the uniform TLO FBMC format is derived by applying the time-frequency duality, formally be replacing symbols $\hat{G}, \omega, F, \tilde{T}$ from (3.5) by $g, t, \omega$ and $\frac{2\pi}{T}$ respectively, (3.5) gets converted to
\[ g_{n,k}(t) = w(t - kT') \exp\left(j \frac{n\pi}{T'} t + j \varphi_{n,k}\right) \]  

(3.9)

As a dual form, \( \{g_{n,k}(t)\} \) also satisfies the orthogonality condition given by (3.10) and thus forms the uniform TLO orthogonality base.

\[
\text{Re}\left\{ \int_{-\infty}^{\infty} g_{n,k}(t)g_{m,l}^*(t)dt \right\} = \begin{cases} 
1 & k = l, n = m \\
0 & \text{otherwise}
\end{cases}
\]

(3.10)

Based on the time-frequency duality between FLO and TLO, expression (3.8) for nonuniform FLO apply as well to the non-uniform TLO FBMC case, with appropriately exchanged variables. Derivation is given below only for the simplest case, where one subchannel has the doubly longer signalling interval of its adjacent one, as illustrated in Figure 3-3.

![Figure 3-3](a) Uniform configuration and (b) its corresponding non-uniform configuration for two subchannels

If the referent impulse response of subchannels marked by 1, 0 and 2' are \( g_{n-1,k-1}(t), g_{n,k}(t) \) and \( g_{m,l}(t) \), and the impulse response of the doubly wide subchannel \( 1' \) is \( g_{s,r}(t) \), the process of time-domain aggregation is expressed through

\[
g_{s,r}(t) = \sqrt{\frac{2}{g_{n-1,k-1}(t) + g_{n,k}^2(t)}}
\]

(3.11)

Going from (3.1) and by placing the newly generated impulse responses (3.11) into the above expression, it follows.

\[
\text{Re}\left\{ \int_{-\infty}^{\infty} g_{s,r}(t)g_{m,l}^*(t)dt \right\} = \text{Re}\left\{ \frac{\sqrt{2}}{\int_{-\infty}^{\infty} g_{n-1,k-1}(t)+g_{n,k}^2(t)g_{m,l}^*(t)dt} \right\} = \\
= \text{Re}\left\{ \frac{\sqrt{2}}{\int_{-\infty}^{\infty} g_{n-1,k-1}(t)g_{m,l}^*(t)+g_{n,k}^2(t)g_{m,l}^*(t)dt} \right\}
\]

Because of absence of overlapping among the uniform TLO subchannels’ impulse responses separated by (at least) one shortest impulse response, the product \( g_{n-1,k-1}(t)g_{m,l}^*(t) \) is equal to 0, so that the orthogonality criterion reduces to the one corresponding to the uniform FBMC case.
\[ \text{Re}\left\{ \int_{-\infty}^{\infty} g_{s,r}(t) g_{m,l}^*(t) \, dt \right\} = \text{Re}\left\{ \int_{-\infty}^{\infty} g_{n,k}(t) g_{m,l}^*(t) \, dt \right\} = \begin{cases} 1 & k = l, n = m \\ 0 & \text{other} \end{cases} \quad (3.12) \]

since the same expression is produced as in equation (3.10), from which we started.

### 3.2 Simulation results in the context of Extended OFDM

![Simulation results for uniform FLO FBMC 100% roll off factor M=32,K=4](image1)

Figure 3-4: Simulation results for uniform FLO FBMC 100% roll off factor M=32,K=4

![Simulation results for uniform TLO FBMC 100% roll off factor M=32,K=4](image2)

Figure 3-5: Simulation results for uniform TLO FBMC 100% roll off factor M=32,K=4

![Simulation results for non-uniform TLO FBMC M=64,K=4](image3)

Figure 3-6: Simulation results for non-uniform TLO FBMC M=64,K=4

![Simulation results for non-uniform FLO FBMC M=64,K=4](image4)

Figure 3-7: Simulation results for non-uniform FLO FBMC M=64,K=4
Figure 3-8: Simulation results for non-uniform FLO FBMC with asymmetrical subchannel spectra M=64,K=4
4. Implementation framework

4.1 Fast Convolution Filter Bank Approach

One of the core ideas of EMPhAtiC is to investigate the feasibility of a special implementation scheme for multirate filter banks which is based on fast-convolution (FC) processing. The basic idea of fast convolution is that a high-order filter can be implemented effectively through multiplication in frequency domain, after taking DFT’s of the input sequence and the filter impulse response. Eventually, the time-domain output is obtained by IDFT. Commonly, efficient implementation techniques, like FFT/IFFT, are used for the transforms, and overlap-save processing is adopted for blockwise processing long of sequences [37]. The application of FC to multirate filters have been presented in [29, 23], and FC implementations of channelization filters/filter banks has been considered in [9, 30, 34, 39]. The idea of FC-implementation of nearly perfect-reconstruction filter bank systems has been introduced in [41]. The frequency spreading FBMC approach of [31] can be seen as a very special case of FC-FB where the IFFT size is two times the overlapping factor of the polyphase FB design, and only one sample is utilized from each IFFT block. Also the variant of Generalized frequency division multiplexing (GFDM) presented in [36] is similar to the FC-FB approach. In this section we report detailed analysis and optimization methods for FC-FB based on [25].

These papers demonstrate the greatly increased flexibility and efficiency of FC-FB in communication signal processing, in comparison with the commonly used polyphase implementation structure. FC-FB provides also means for implementing efficiently non-uniform filter bank structures, also with asymmetric spectral shaping.

In Section 4.2.1, a model for multirate FC processing is first developed. This model makes it possible to efficiently analyze and optimize the frequency-domain characteristics of FC-FBs. Next in Section 4.2.2, various intricate aspects of FC multirate processing are discussed and analyzed, and potential applications in communications signal processing are highlighted. In Section 4.2.3, design examples are presented, including root-raised-cosine (RRC) and raised-cosine (RC) type FBs, as well as arbitrary transition band cases. The flexibility of the FC-FB approach in tuning the subband bandwidths and center frequencies is highlighted. Comparisons between direct RRC/RC solutions and optimized designs are presented. Also the computational complexity of FC-FB’s is evaluated and compared against the commonly used polyphase implementations in case of NPR FB systems. In Section 4.2.4, a case study of FC-FB design, with parameters of the 5 MHz LTE system, is presented. The resulting spectral characteristics are presented, the implementation complexity is compared with basic OFDM/OQAM scheme, and multimode parameterization aspects are discussed. For the demonstrator development purposes, a detailed case study with 1.4 MHz LTE parameters is presented in Section 4.2.4.
Figure 4-1: Overlap-save processing for fast convolution including the notations used for
the number of samples in different parts of the overlap-save blocks. The subscript \( k \) is used
only in the context of non-uniform filter banks. For clarity of notation, we assume most of
the time that the overlapping parts are symmetric both on the high-rate and low-rate sides.
However, this is not an essential assumption, and in the discussions of Section 4.2.2, the
need for non-symmetric overlap on the low-rate side becomes apparent.

4.1.1 Fast-Convolution Processing

In the basic form, FC implements linear convolution between two finite-length
sequences as follows:

\[
c[n] * x[n] = \text{IFFT}(\text{FFT}(c[n]) \cdot \text{FFT}(x[n])).
\]  

Assuming that the lengths of the sequences are \( N_c \) and \( N_x \), the length of the convolution
is \( N = N_x + N_c - 1 \). The lengths of the FFT and IFFT operations in (4.1) have to be at least \( N \),
otherwise cyclic distortion will appear in the result. In order to process long sequences,
blockwise FC can be used in conjunction with overlap-add or overlap-save processing [37].
We use the latter one because of its better properties in finite wordlength implementation.
This is due to strong transients at the beginning and end of each overlap-add processing
block. These transients often greatly exceed the numeric range of the final output signal
while the transients of consecutive overlapping blocks are canceling each other.

Let us assume that in (4.1), \( c[n] \) represents a finite-length linear filter impulse
response, and that we want to use the block length of \( N \), with \( N > N_c \), in the FFT
processing. Since symmetric prototype impulse responses are generally considered, we
utilize symmetric (zero-phase) model for overlap-save processing, illustrated in Figure 4-1.
4.1.1.1 Multirate Filtering and Filter Banks

The structure of the proposed variable analysis filter bank (AFB) is illustrated in Figure 4-2. The general idea of the structure is a multirate version of fast convolution [23, 4]. We consider a case where the incoming high-rate, wideband signal is to be split into several narrowband signals with adjustable frequency responses and possibly adjustable sampling rates. We are interested in cases where the output signals are critically sampled or oversampled by a small factor. We also note that different subbands may be overlapping. The dual structure of Figure 4-2 can be used for combining multiple low-rate, narrowband signals into a single wideband signal, following the frequency-division multiplexing principle.

Figure 4-2 includes sampling rate reduction by factors

\[ R_k = \frac{N}{L_k} = \frac{N_S}{L_{S,k}} \]

(4.2)

where \( k \) is the subband index and the notations used for the lengths of the overlap-save processing blocks is indicated in Figure 4-1. Given the FFT length \( N \), the sampling rate conversion factor is determined by the IFFT length, and can be configured for each subband individually. Naturally, the IFFT length determines the maximum number of non-zero frequency bins, i.e., the bandwidth of the subband. It is also possible to increase the subband output sampling rate by increasing the IFFT length by adding zero-valued bins outside the wanted subband frequency range. In communication applications, a subband would contain a communications waveform with specific symbol rate, and the output sampling rate is normally chosen as its (small) integer multiple.

Regardless of the specific processing applied for each subband, the decimation factor of the structure of Figure 4-2 is given by (4.2) [8]. This means that the length of the FC output block is reduced by the same factor, and so is also the length of the overlapping part in the overlap-save processing. The input and output block lengths have to exactly match,
taking into account the sampling rate conversion factor. It follows that in (4.2), both \( \frac{N}{L_k} \) and \( \frac{N_s}{L_{S,k}} \) have to be integers. Consequently, \( L_k \) has to be a multiple of \( \frac{N}{\text{gcf}(N, N_s)} \) where \( \text{gcf}(\cdot) \) stands for the greatest common factor. For example, if \( N_s = 3N/4 \), \( L_k \) has to be a multiple of 4, or if \( N_s = 4N/5 \), \( L_k \) has to be a multiple of 5. If \( f_s \) is the input sampling rate, the possible output sampling rates are multiples of \( 4f_s/N \) and \( 5f_s/N \), respectively. So the configurability of the output sampling rate depends greatly on the choice of \( N \) and \( N_s \).

There are two key parameters which have an effect on the spectral characteristics of the FC-FB scheme:

- The IFFT size \( L \) is defining how well the filter frequency response can be optimized. In general, increasing the value of \( L \) helps to improve the stopband attenuation.
- The overlap factor \( 1 - L_s/L \) : In FC based multirate signal processing there is an inevitable cyclic distortion effect because the overlapping part of the processing block cannot be made big enough to absorb the tails of the filter impulse response. Naturally, this effect can be reduced by increasing the overlap factor.

Next these effects are analyzed using a periodically time variant model for FC processing and effective tools for frequency response analysis and FC filter optimization are developed.

![Figure 4-3: FFT-domain weights for RRC-type subband processing with roll-off of 1.](image)

**4.1.1.2 Signal Model for Multirate FC Processing**

In the following, we will develop a signal model for analyzing the most central impairment of FC-based multirate systems. Without loss of generality, we focus on the single-channel lowpass decimation case, and the subband index \( k \) is excluded from the block-length expressions.

Targeting at efficient implementation, only passband and transition band frequency bins take non-zero values in FFT-domain processing, as illustrated in Figure 4-3. Then the corresponding filter impulse response becomes quite long and it is unrealistic to use overlapping periods which would be able to absorb the impulse response tails. The resulting problem is illustrated in Figure 4-4: Since FC processing implements cyclic convolution, cyclic distortion will appear in the impulse response, especially for the first and last samples of a data block. Figures 4-4 (b) and (d) illustrate the equivalent impulse responses for linear convolution, when the cyclic distortion takes place. We notice that in the presence of cyclic distortion, the linear system model is time-varying, i.e., the effective impulse response is different for different output samples within the data block.

Even though FC processing does not directly utilize convolution processing, the model illustrated in Figure 4-4 is still a valid model for characterizing the cyclic distortion
effects. FC is a precise implementation of cyclic convolution, as long as effects due to limited numerical precision are not under consideration.

Let $h[\eta]$ denote the target linear impulse response and let $\tilde{h}_n[\eta]$ denote the effective linear impulse response for output sample $n$ when cyclic distortion takes place. Assuming that the filter impulse response length is $2N_F + 1$, this can be expressed formally as follows:

**Case 1:** $0 \leq n < N_F - N_O$

$$\tilde{h}_n[\eta] = \begin{cases} 
  h[\eta + N_S] & \text{if } n-N_S+N_O < \eta \leq N_F - N_S \\
  h[\eta] & \text{if } -N_F \leq \eta \leq n + N_O \\
  0 & \text{otherwise.}
\end{cases} \quad (4.3a)$$

**Case 2:** $N_F - N_O \leq n < N_S - N_F - N_O$

$$\tilde{h}_n[\eta] = h[\eta]. \quad (4.3b)$$

**Case 3:** $N_S - N_F - N_O \leq n < N$

$$\tilde{h}_n[\eta] = \begin{cases} 
  h[\eta + N_S] & \text{if } n-N_S+N_O < \eta \leq n + N_f \\
  h[\eta - N_S] & \text{if } N_S - N_F < \eta \leq n + N_O \\
  0 & \text{otherwise.}
\end{cases} \quad (4.3c)$$

Next we focus on developing a tractable model for the time-variability. Since this is characterized by time-varying impulse response, such a model is difficult to establish for a generic input signal. Therefore, we first focus on the case of single-tone (complex exponential) input signal

$$x[n] = C e^{j\omega n}. \quad (4.4)$$

Now the $n$th output sample can be obtained using convolution as follows

$$y[n] = \sum_\eta \tilde{h}_\eta[x[n-\eta]] \quad (4.5a)$$

$$= C e^{j\omega n} \sum_\eta \tilde{h}_\eta[e^{-j\omega n}] \quad (4.5b)$$

$$= C e^{j\omega n} \tilde{H}_n(e^{j\omega}) \quad (4.5c)$$

where $\tilde{H}_n(e^{j\omega})$ is the discrete-time Fourier transform of $\tilde{h}_\eta[\eta]$.

We can see that each output sample of a data block is multiplied by a complex coefficient which corresponds to the Fourier transform of the corresponding equivalent impulse response at the frequency of the input tone. In general, these complex coefficients are different for different samples $n$ within the output data block. Thus, for a single input tone, the cyclic distortion can be modelled as a modulation by a periodic sequence which depends on target impulse response and the parameters of FC processing [22, 19].
Figure 4-4: Illustrating the cyclic distortion effects and equivalent linear impulse responses when block overlap is smaller than impulse response length. (a), (b) For first samples of the data block. (c), (d) For last samples of a data block.

For a non-decimating filter, the length of the modulating sequence is equal to the non-overlapping data block length. It is also easy to see that when the input signal period is equal to the overall block length $N$ or its submultiple, the cyclic convolution is equal to linear convolution, and no distortion appears. For those frequencies, the frequency response comes directly from the FFT-domain weights. Notably, the FC-based filter has zeros at frequencies corresponding to the FFT bins in the stopband region.

When FC processing is used for decimation by a factor of $R$, the period of the modulating sequence is $L_S = N_S/R$ output samples. The modulating sequence can be expressed as

$$\tilde{H}_{b^+(l,R)_{LS}}(e^{j\omega})$$

where $l_0$ is the index of the first sample of the output data block within the input data frame and $(l)_I$ denotes $l$ modulo $I$.

As a periodic sequence, the modulating sequence can be expressed through a Fourier series consisting of frequencies $2\pi k/L_S$. Then the modulation creates frequencies $f_o + 2\pi k/L_S$, $k = 0, 1, \ldots, L_S$, where $f_o$ is the principal output frequency corresponding to the input tone after the spectral folding due to sampling rate reduction. Thus we see that the modulation effect caused by cyclic distortion splits the energy of a tone to multiple frequencies within the output Nyquist range. The magnitude of the principal frequency component is proportional to the mean value of the modulating sequence. The total power
of the other components, referred to as modulated components, is proportional to the variance of the modulating sequence.

Now we have established a model for characterizing the effect of cyclic distortion on tones at arbitrary frequencies. Our model for FC processing is periodically time-varying but linear, an LTV model, and therefore the superposition principle still applies. The effect of cyclic distortion on an arbitrary input signal can, in principle, be obtained by expressing the input signal as a sum of tones and analyzing the effects on each of the tones. In practical applications, for a significant tone at the passband region, the principal component dominates in the output, and the modulated components appear as noise. Next we consider practical examples of the developed FC signal model.

![Figure 4-5: Time-domain simulation results in the presence of two tones in the input of a FC decimator, either separately or together.](image)

**4.1.1.3 Numerical Examples**

The numerical examples below are based on the following parametrization: \( N = 1536 \), \( N_s = 3N/4 = 1152 \), \( R = 96 \), \( L = 16 \), and \( L_s = 12 \).

First we check the output signal when the input consists of two tones in the filter stopband region. Figure 5 shows the output spectrum when each of the tones is present alone, as well as the case where they are present at the same time. For both input tones, we see \( L_s = 12 \) equally spaced tones in the output, as predicted by the developed model. Comparing the simulations where the tones are present separately or simultaneously, we can observe perfect match, implying the validity of the superposition principle.

Figure 4-6 shows the frequency response of the lowpass filter. Since we are considering a time-varying multirate system, the definition of the frequency response is not quite straightforward. Therefore, the frequency response has been obtained by measuring the signal power at the subband output in the presence of sinusoid at the frequency of the horizontal axis. Further, in many of the figures only the local maxima of the frequency responses are plotted, and the stopband notches are not visible. Such an envelope plot is used in the main part of Figure 4-6 for the sake of clarity, whereas the zoom part shows
complete frequency responses. In this figure, the principal and modulated components are shown separately. From the figure we can see that for sinusoids in the stopband region, the output is dominated by the modulated components, whereas for tones in the passband region, the modulated components are at a relatively low level.

![Figure 4-6: Frequency response of the lowpass decimating filter, separately for the principal and modulated output tones.](image)

### 4.1.1.4 Frequency Response Calculation

The FC filter bank design process presented below is based on the FFT-domain model of the lowpass prototype filter, since this model leads to the minimum number of free parameters in the optimization. The definition of frequency response is based on the total interference at a subband output due to a tone at the analysis filter bank input. Based on the multirate FC processing model presented above, the process for evaluating the frequency response can then be summarized as follows:

1. Assume that \( N, L, \) and \( L_s \) are given, together with \( L \) frequency bin values.
2. Calculate the impulse response \( h(\eta) \) using IFFT.
3. Construct \( \tilde{h}_n(\eta) \) using (3) for the selected set of \( L_s \) resampling phases \( n \in \Lambda \).
4. Calculate \( \tilde{H}_n(e^{j\omega_i}) \) using FFT for \( n \in \Lambda \) and for the set of frequencies \( \omega_i \in \Omega \) used in the analysis or design.
5. Calculate the total interference values for the used set of frequencies as \( \sum_{n \in \Lambda} \left| \tilde{H}_n(e^{j\omega}) \right|^2 / L_s \) for \( \omega_i \in \Omega \).
4.1.2 Enhancements to the Fast-Convolution Multirate Filter Model

4.1.2.1 Sampling Phase Control

Following the common model of time-domain decimation filters consisting of a digital filter stage and a down-sampling stage [33], Figure 4-7 shows non-decimated filtered processing blocks out of which the decimated output blocks are obtained through uniform subsampling. In practical implementation of a FC-based decimator, such non-decimated blocks are not available, but the model is valid and useful for various analytical purposes. One important practical issue highlighted by the model is the choice of the sampling phase in the down-sampling operation.

There are two reasons for paying attention to the down-sampling phase:

- In various applications (e.g., communications receiver signal processing) the sampling phase should be synchronized to the input signal.
- The sampling phase has an impact on the cyclic distortion introduced by the FC processing.

For decimation factor $R$, there are $R$ different choices for the down-sampling phase. Figure 4-4 is based on zero-phase model for the filter impulse response, which in this discussion is assumed to be symmetric.\(^2\) This leads to delay-free model for the processing of the non-overlapping sample blocks. Consequently, this processing model corresponds to the non-symmetric impulse responses, but then the filtering delay is not uniquely defined.
earliest possible down-sampling phase. In other words, the first sample of the output save block is the first sample of the non-overlapping part of the corresponding non-decimated filter output block.

The symmetry of resampling depends on the parametrization of the filter bank, as illustrated in the example of Figure 4-8. We can observe that (near) symmetry of the impulse responses can be reached for odd values of \( L - L_S \) by earliest sampling phase, but not for even values of \( L - L_S \).

![Diagram](FFT block: \( L_S/L = 7/8 \)

1 1 1 1 1 1 1 1

IFFT block:

6/8 2 2 2 2 2 2 2 2

5/8 2 2 2 2 2 2 2 2

Figure 4-8: Example of earliest resampling (dashed) versus nearly symmetric resampling (solid) with FFT length of \( N = 32 \), IFFT length of \( L = 8 \) and different output block lengths \( L_S \). With odd \( L_S \), earliest resampling is also nearly symmetric.

One obvious possibility for synchronizing the sampling phase to the input signal is through the initial block buffering. The alternative approach is to adjust the filtering delay, which can be modelled by delaying/advancing the filter impulse response in the illustration of Figure 4-4. The output samples are spaced at \( R \) (decimation factor) input samples, the first one at reference time 0 and the last one at time \( N_S - R \). Advancing the impulse response by \((R-1)/2 \) samples would lead to near symmetry of the resampling instances in case of even \( L_S \). An advance of \( \tau \) input samples in the filter impulse response corresponds to introducing an additional linear phase response in the FFT-domain:

\[
H_k^{(\tau)} = H_k e^{j2\pi \tau/N}
\]  

where \( H_k \), \( k = 0,1,\ldots,N-1 \) is the FFT of the zero-phase filter impulse response. Thus the filter phase slope can be used for controlling the sampling phase in the FC processing. It should be noted that when using a zero-phase filter (with symmetric impulse response), the FFT-domain weights are real, whereas the sampling phase control leads to complex weights. This has an impact on the computational complexity, as discussed later.

Regarding the cyclic distortion, it can be expected that it would be advantageous to make the down-sampling as symmetric as possible within the data blocks. For odd decimation factors fully symmetric sampling can be reached, for even decimation factors approximate symmetric. Figure 4-9 shows examples of the modulation sequences of the LTV model for non-symmetric and approximately symmetric (with half input sample offset) down-sampling cases, considering sinusoids both in passband and stopband. Here the system parametrization is the same as in the numerical examples above. Due to even \( L_S \), earliest resampling is not symmetric. We can see that the near symmetric sampling phase, obtained through the phase response adjustment, results in near symmetry of the
modulation sequence and improved interference suppression. In this case the total interference is reduced by about 5 dB for the passband tone and about 2 dB for the stopband tone.

While optimizing the interference effects through the choice of the sampling phase, the overall filter delay can be controlled by the input buffering. As an example, if an advance of $R/2 - 1$ (the near symmetric case for even $L_s$) is introduced through the phase slope in FFT domain, then the zero-delay processing model is reached by delaying the block buffering by the same number of samples. The input sample with time index 0 should be preceded by $N_o + R/2 - 1$ earlier samples and/or zero padding, in order to initialize the block buffering properly.

![Figure 4-9: Effect of the sampling phase on the magnitude of the modulation sequence in the LTV signal model with $N = 1536$, $L = 16$, and $L_s = 12$. (a) Passband tone with earliest sampling phase. (b) Stopband tone with earliest sampling phase. (c) Passband tone with near symmetric sampling phase. (d) Stopband tone with near symmetric sampling phase. The tones are at the second passband interference maximum and at the first stopband sidelobe maximum in Figure 4-5.](image)
It is useful to notice that when the phase slope of the FFT-based filter is used for synchronizing purposes, also time-shifts by non-integer number of input samples can be easily implemented.

Summarizing this discussion from the multirate FC filter design point of view: If the signal sampling phase is insignificant for the application, or the synchronization is done outside the FC processing structure, then the optimum sampling phase from the interference minimization point of view can be utilized. However, especially in communications receiver signal processing, the flexibility of the FC processing structure can be used also for compensating the sampling time offsets, and in this context the interference level with the worst-case sampling phase might be the relevant metric. We shall continue with this topic in Section 4.2.3 from the multirate FC filter optimization point of view.

4.1.2.2 Processing Bandpass signals

So far our analysis has focused on lowpass decimation filters, but the developed models can be readily applied also to bandpass decimation filters and filter banks. For bandpass filters, the FFT-domain weights are obtained by shifting the lowpass filter weights by an integer number of FFT bins to appear around the bandpass center frequency $f_c$. Following the usual spectral folding model for complex I/Q signals, a spectral component at input frequency $f_c$ will appear at frequency $f_c^{(d)} = \text{mod}(f_c, f_s/R)$ in the frequency range $[0, f_s/R]$, or equivalently, in the range $[f_s/(2R), f_s/(2R)]$. In time-domain bandpass subsampling, a symmetrically located lowpass signal is obtained if $f_c = k f_s/R$, i.e., the center frequency is an integer multiple of the output sampling rate. In time-domain processing, the spectrum of the decimated signal can be shifted to the desired position by mixing, i.e., multiplication by the complex exponential $e^{-j2\pi k f_s^{(d)} / f_s}$.

In the FC multirate filter model for bandpass signals, a frequency shift by an arbitrary number of FFT bins is directly introduced. The frequency shift is defined by the center of the FFT-domain weights. The symmetry of the weights is not a fundamental requirement, and the translation is defined by the FFT bin index which is connected to the DC bin (bin 0) at the IFFT input.

However, there is one important issue which must be addressed in this context [23]. As different FFT blocks are processed independently in the FC multirate structure, the processing of consecutive blocks does not necessarily follow the time-domain complex mixing model, as it should. Effectively, the mixing sequence for each IFFT block is

$$c_{\text{mix}}(l) = e^{-j2\pi k_0 f_s^{(d)} / f_s} = e^{-j2\pi k_0 l / L}$$

for $l = 0, 1, ..., L_s - 1$, where $k_0$ is the center bin of the bandpass filter. If the bandpass center frequency is a multiple of the output sampling rate, i.e., $k_0$ is a multiple of $L$, then this mixing sequence is all 1’s and the lowpass processing model applies. Otherwise, the mixing sequence is non-trivial and, more importantly, there are phase discontinuities of the sequence between blocks, which is not in agreement with the time-domain mixing model. Proper phase continuity of the mixing sequence is obtained by introducing blockwise phase rotation.
where $b$ is the block index. So all the samples of the $b$th output block should be multiplied by $c_{pb}(b)$.

### 4.1.2.3 Filter Bank Models for Communications Signal Processing

The structure of Figure 4-2, together with its dual synthesis bank structure, can be used for implementing separate analysis and synthesis filter banks, as well as for analysis-synthesis or transmultiplexer (TMUX) type FB configurations. Our focus is on complex-valued (I/Q) time-domain signals commonly applied in communications signal processing.

Analysis and synthesis filter banks find applications in channelization filtering for software defined radio receivers and transmitters, respectively. It has been observed that FC-FBs are particularly useful in this application since the channel bandwidths and center frequencies can be easily and rapidly adjusted during real-time operation [24, 35]. The basic non-overlapping complex FC-FB scheme is feasible for this application, and raised-cosine (RC), root-raised-cosine (RRC), or arbitrary transition band designs may be considered.

However, in systems where specific pulse shaping filtering is defined, it is possible to combine the (typically) RRC-type transmitter or receiver pulse shaping filtering with the channelization function. Typically subchannel sample sequences with oversampling by a small integer factor is needed for digital baseband processing. As a generalization of this idea, FC-FB based receiver structure can be used for simultaneously processing multiple communication signals of different waveform types [34], bandwidths, and other characteristics:

- Linear digital modulation (e.g., QAM): channelization and RRC filtering.
- GMSK and other FSK-type modulations: channelization.
- CDMA signals: channelization and chip-level RRC filtering, together with narrowband interference suppression.
- OFDM signals [41]: channelization, also in case of non-contiguous spectrum use, together with narrowband interference suppression.
- Filtered multitone (FMT) type multicarrier waveforms [15], which use non-overlapping QAM-modulated subcarriers: FMT processing, including channelization and RRC filtering, can be implemented using FC-FB with non-overlapping IFFTs
- The FC-FB structure can also be used for implementing filter bank systems with overlapping nearly orthogonal channels. Among a few alternatives regarding the modulation format and subchannel staggering, we use here the offset-QAM (OQAM) signal model [38, 27]. We refer to this waveform as FBMC/OQAM (in the literature also OFDM/OQAM).

Here we focus on the FBMC/OQAM and FMT cases, while single carrier QAM transmission can be considered as an extreme case of FMT. In our approach, the basic design is done for a filter channel with roll-off of one. The frequency-domain weights consist of two symmetric transition bands and all stopband bins are set to zero as illustrated in Figure 4-3. Figure 4-10 shows an FBMC/OQAM type multiplex of subchannels, which is constructed using such basic filters. The subchannel spacing is half of the overall subchannel
bandwidth, which is equal to the IFFT length on the AFB side. One important feature of the FC-FB structure is that the transition band shape optimized for the basic case can be used for constructing filters with arbitrary bandwidths. In Figure 4-10(b) a filtered multi-tone (FMT) type multiplex of non-overlapping subchannels is shown and Figure 4-10(c) shows a single-carrier transmission channel, both using the transition band shapes of the basic design. The subchannel bandwidths and center frequencies can be independently tuned, with the resolution of the FFT bin spacing. Different types of multiplexes can be combined in a single FC-FB structure. It is also possible to use different transition band shapes for different multiplexes, if so desired.

Due to the distortion effects, FC implementation of perfect reconstruction filter banks is probably not worth considering, but nearly perfect reconstruction (NPR) FB’s with controlled level of distortion effects are feasible. For NPR-FBs with complex subchannel signals, special signal and FB models have to be adopted. In other words, with basic I/Q signal models for the subchannel signals, it is not possible to reach NPR characteristics. First of all, it is a necessary requirement that the subband signals are oversampled (usually) by a factor of two. In the FBMC/OQAM case, the I/Q modulated signal in each subchannel appears in such a way that the real parts of the symbols are in subchannel samples with even time-indices, whereas the imaginary parts appear with odd indices, or vice versa. Furthermore, the pattern of real and imaginary parts has to be alternating between even/odd subchannels, e.g., real parts at even time indices in even subchannels, and at odd indices in odd subchannels. Following this model, the implementation of NPR-FB systems with the structure of Figure 2 is straightforward, and uncompromised performance has been verified through simulations.

One fundamental issue in uplink transmission using multicarrier techniques is that the different users’ signals (groups of subchannels) have to be carefully synchronized with each other, which leads to the requirement of tight base-station control and fairly complicated synchronization procedures. In FB based uplink transmission, in contrast to OFDMA [41], different users’ groups of subchannels are commonly assumed to be separated by at least small guardbands. Therefore, different users subcarrier groups are not significantly overlapping within the expected range of carrier frequency offsets. Then, using the structure of Figure 4-2, different users’ frequency offsets can be compensated independently of the others through the selection of the FFT bins on the receiver side. This can be done in such a way that the residual frequency offsets are no more than half of the FFT bin spacing. Combining this with the subcarrier-wise synchronization ideas of [28],
significant relative frequency offsets, as well as arbitrary timing offsets, can be compensated in FB based uplink transmission.

4.1.3 FC Filter Bank Design

Here we focus mostly on RRC type filters, considering later also RC type filters [34], as well as arbitrary transition band cases. RRC-type filters can be used as prototypes for complex NPR filter banks for FBMC/OQAM transmultiplexer systems, and the other types find applications, e.g., as channelization filters. The FFT bins of the filter transition bands can be obtained readily by sampling half a cycle of the cosine function (RC case) or its square root (RRC case). However, also this approach suffers from cyclic distortion and is not optimal in any sense. Therefore, we consider ways to design the FFT-weights in order to optimize the performance. The primary optimization criterion is stopband attenuation. In our designs, relatively low number of non-zero FFT bins is utilized for each subband. Imposing strictly the RRC or RC type symmetry further reduces the number of free variables in the optimization. This allows us to use generic optimization algorithms to minimize a stopband frequency response based cost function.

![Figure 4-11: Example of frequency bin symmetry in RRC-type design with $L = 12$.](image)

4.1.3.1 Optimization Principles

As the basic design, we consider cases with the roll-off factor of one. The IFFT length $L$ is assumed to be even. Then it follows that each subband has $L/2$ non-zero frequency bins, see Figure 11 for an example. The center bin (at DC in the lowpass filter case) takes the value 1 and, when $L$ is a multiple of 4, the two bins in the middle of the transition bands take the value $1/2$ for RC cases and $1/\sqrt{2}$ in RRC cases. In the RC case, each pair of bins located symmetrically around in the transition band add up to one, and in the RRC case their squares add up to one. Consequently, in both cases there are $\left\lfloor L/4 \right\rfloor - 1$ independent parameters in the optimization. In the following, we consider IFFT lengths of 8, 12, 16, and 24 with 1, 2, 3, and 5 free parameters, respectively. In the arbitrary transition band case, there is only symmetry of the two transition bands, but no constraints on the transition band shape. Consequently, there are $L/2 - 1$ independent parameters.
Figure 4-12: Impulse response truncation effects for $N = 1536$, $L = 12$, and $L_S = 7$.
(a) Direct RRC design with earliest, near symmetric resampling, SPR=(44.8, 81.0) dB.
(b) Direct RRC design with delayed non-symmetric resampling, SPR=(48.7, 118.5) dB.
(c) Optimized RRC-type design with $\text{SPR}_c$ criterion and earliest resampling, SPR=(57.7, 95.8) dB.

To define the cost function for optimization and comparison of different designs, we consider a situation where a single active subchannel is present in the analysis bank input. This subchannel has RRC or RC spectrum, depending on the design case. The cost function is the overall power leaking to a specific FC-FB subband. Without loss of generality, the interference is measured at the lowpass (DC) subband. We consider two cases: (i) The active subchannel is channel 2, i.e., the second adjacent subchannel. This is the closest subchannel fully in the stopband of the observation subband. (ii) Most distant subchannel, close to half of the sampling rate. These metrics are referred to as second subchannel power ratio, $\text{SPR}_2$, and distant subchannel power ratio, $\text{SPR}_d$. In the following we consider the optimization of FC filter banks for either of these criteria.

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3 This is the case for the FBMC/OQAM system, but we use the same notation also for other cases, i.e., $\text{SPR}_c$ refers to the subchannel at the distance of $L$ bins from the reference subband.
Figure 4-13: Frequency response for $\text{SPR}_2$-optimized RRC-type design with symmetric resampling and $N=1536$, $L=12$, and $L_S=7$. Also frequency responses for polyphase filter banks with $K=3$ and $K=4$ are shown.

Figure 4-14: Subchannel protection ratios of RRC-type designs as a function of the overlap factor for $N=1536$ and (a) $L=8$, (b) $L=12$, (c) $L=16$, and (d) $L=24$. 
The resulting stopband characteristics are expressed through the \( \text{SPR}_2 \) and \( \text{SPR}_d \) values. To express this pair of numeric values, we use a compact notation, e.g., \( \text{SPR}=(44.8, 81.0) \) dB. The design procedure can be easily modified for minimizing the energy in the whole stopband or any part of it, with selected weight function.

The filter bank optimization is based on calculating the total interference for a sufficiently dense grid of frequencies in the frequency band selected for the cost function calculation. This can be done using the effective procedure of Section 4.2.1.4. In our designs, we used 16 frequency points between consecutive notches in the stopband, i.e., \( 16L \) frequency points are used for the \( \text{SPR}_2 \) and \( \text{SPR}_d \) criteria. The calculation of the cost function takes a fraction of a second and a single filter bank optimization takes a few seconds on a standard PC computer.

Considering the transform lengths for FC-FB’s, it is clear that using only power-of-two length for FFT length would severely restrict the system parametrization. This is because also the IFFT length would then have to be a power of two. For this reason, we consider here also transform lengths of the form \( 3\hat{N} \) where \( \hat{N} \) is a power of two.

### 4.1.3.2 Designing RRC-Type Filters

Let us now focus on RRC-type filter bank designs. For 100 % roll-off, the directly constructed RRC filter’s impulse response has a special structure, which can be seen in Figure 4-12. The impulse response resulting from these designs has exact zero crossings at multiples of \( N/L = 128 \) samples, except for the two locations within the main lobe.\(^4\) This property helps to understand certain effects of the sampling phase control discussed in Section 4.2.2.1. It happens that with odd lengths of the output block \( L_S \) and earliest resampling, providing near symmetry, the impulse response is truncated in the middle of a sidelobe. This effect, degrading the frequency response severely, is illustrated in Figure 4-12 for an overlapping factor of 5/12. In this case, significantly better stopband attenuation can reached through resampling delayed by \( R/2-1 \) samples. Figure 4-12(c) shows also the corresponding optimized design, avoiding the mentioned effect with symmetric resampling and resulting in superior stopband attenuation in comparison to the direct RRC designs. Figure 4-13 shows the frequency response of the \( \text{SPR}_2 \) optimized filter of Figure 4-12(c) with \( \text{SPR}=(57.7, 95.8) \) dB, together with the frequency responses of polyphase filter banks with \( K = 3 \) and \( K = 4 \) [43]. Here \( K \) defines the impulse response length as \( KM \) where \( M = 2N/L = 256 \) is the number of subchannels. The \( \text{SPR}_2 \) is 50.0 dB and 62.7 dB with \( K = 3 \) and \( K = 4 \), respectively. It can be seen that the polyphase filters’ stopband attenuation increases heavily with increasing distance from the active subchannel, whereas in the FC design, the attenuation saturates at a relatively high level. Optimizing the FC filter bank with the same parameters for the \( \text{SPR}_d \) criterion results in \( \text{SPR}=(56.4, 104.9) \) dB, i.e., slightly increased distant attenuation with somewhat reduced \( \text{SPR}_2 \).

In order to get the following results, we have considered both earliest and delayed (by \( R/2-1 \) samples) resampling phases, and picked up the one which minimizes the cost function. It has been observed that with higher overlapping factors, the behavior between these extreme cases is usually monotonic. However, with small oversampling factors, this is

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\(^4\) For 100 % roll-off, the RRC spectrum is the same as the spectrum of the duo-binary pulse [13], consisting of two sinc pulses with the spacing of 128 samples in our example case.
not necessary the case, and the cost function could be improved somewhat with suitable intermediate choices. These cases are not included in the results below as they are not likely to be useful in practical applications. Figure 4-14 shows the performance of different RRC-type designs as function of the overlap factor for the FFT length \( N = 1536 \) and four different values of \( L \). Figure 4-14 shows the results as a function of the FFT length for four different overlap factors for the IFFT length of \( L = 16 \). Based on these results we can make the following observations about RRC-type FC-FB design:

- Proper choice of resampling phase is important, both for direct RRC and optimized designs.
- For small overlap factors \( L \leq 4/L \), the optimization gives only marginal improvement over properly configured direct RRC designs. On the contrary, with \( \text{SPR}_2 \) criterion, the optimization results in minor improvement of \( \text{SPR}_2 \) with significant loss in \( \text{SPR}_d \). For higher overlap factors, significant improvement can be achieved.
- There is an obvious trade-off between the two criteria. Often relatively small improvement in \( \text{SPR}_2 \) leads to significant loss in \( \text{SPR}_d \).
- \( \text{SPR}_2 \) performance of RRC-type designs is rather independent of the FFT length. \( \text{SPR}_d \) increases with increasing length, i.e., increasing number of subbands \( M = 2N/L \).

We have seen that a FC-FB is a periodically time-varying system, so the distortion effects are time-dependent. This is illustrated in Figure 4-16 which shows the inband and out-of-band distortion effects based on time-domain simulation for a FC-FB/OQAM system with \( N = 4096 \), \( L = 16 \), and \( L_S = 12 \) or \( L_S = 10 \). The system is optimized for \( \text{SPR}_2 \) criterion. Out of the 512 subchannels, 300 contiguous ones are used for transmitting offset-QPSK modulated data. The inband interference is measured over all the active subchannels. It is partly due to the NPR nature of the FB system and partly due to the cyclic distortion of FC-based implementation. Out-of-band power leakage is measured at the 2nd, 4th, and 10th subband above the highest active subchannel. Figure 4-16 clearly shows the increased interference level at the edges of the IFFT block. The inband interference is verified to be at an acceptable level for wireless communication applications.

### 4.1.3.3 RC-Type and Arbitrary Transition Band Designs

Figures 4-17 and 4-18 show the performances of the RC-type and arbitrary transition band designs with similar parameters as in Figure 4-14 for RRC designs. We can make the following conclusions:

- RC designs give consistently clearly better \( \text{SPR}_2 \) performance than RRC designs. However, with certain (low) overlap factors, the \( \text{SPR}_d \) performance of RC is clearly inferior to RRC.
- The optimized RC-type designs show some improvement with the lowest overlap factors, but these cases are not likely to be interesting in practice. For higher overlap factors, significant improvement can be achieved in a similar way as in the RRC case.
- The performance of arbitrary transition band designs clearly exceeds the performance of RC- and RRC-type designs.
Figure 4-14: Subchannel protection ratios of RRC designs as a function of the FFT length for $L = 1536$ and overlap factor (a) $8/16$, (b) $6/16$, (c) $4/16$, and (d) $2/16$.

Figure 4-16: Interference within an IFFT block in an FC-FB/OQAM system with $N = 4096$, $L = 16$, and $L_S = \{10,12\}$. 512 subchannels, 300 active. (a) Inband interference. (b) Out-of-band interference. The optimized SPR is $(51.7, 96.5)$ dB for $L_S = 12$ and $(64.8, 108.9)$ dB for $L_S = 10$. 
It should be noted that RC and arbitrary transition band designs with roll-off of one are not very useful as such. However, they can be used as the basis for constructing channelization filters with somewhat wider bandwidths, and consequently with smaller roll-offs, using the ideas presented below.

### 4.1.3.4 Subchannel Bandwidth Adjustment

One of the central characteristics of FC-FB is the flexibility in adjusting the subband bandwidths. One possible approach is to use the transition band shapes designed for the basic case, with roll-off of one, to construct wider subbands in the FFT domain. For example to double the bandwidth, $L/2$ additional FFT bins of value one are inserted in the passband, and $L/4$ 0-bins are inserted outside transition band regions. The roll-off of the resulting subband becomes 0.5. The same principle can be followed to create subbands with any IFFT length $\hat{L}$ for which $(\hat{L} - L)/4$ is an integer and $N/\hat{L}$ is an integer. The resulting roll-off is $L/\hat{L}$. For example with $N = 1536$ and $L = 12$, the following IFFT lengths may be considered: 16, 24, 32, 48, 64, 96, . . . . Figure 4-19 shows examples with $L = 12$ and $\hat{L}/L$ is 2, 4, or 8.
based on RRC, RC, and arbitrary transition band designs optimized using the SPR$_2$ criterion. We can observe that in these example cases:

- First stopband sidelobe remains close to the original level in all three cases.
- In the RC case, the stopband attenuation characteristics are improved when the bandwidth is widened.
- In the RRC case, the far attenuation is reduced when the bandwidth is widened.
- In the arbitrary transition band case, the stopband characteristics are significantly degraded when the bandwidth is widened.

These preliminary results clearly indicate that subband bandwidth adjustment based on the presented idea is feasible in practice. It is possible to optimize the transition band shape for each value of $\hat{L}$ using the procedure described earlier. However, in practice it would be preferable to use a fixed transition band shape when tuning the bandwidth, in which case the best trade-off for all the needed bandwidths should be sought in the optimization. Detailed investigation of the design trade-offs is left as a topic for future work.

Figure 4-17: Subchannel protection ratios of the arbitrary transition band designs as a function of the overlap factor for $N = 1536$ and (a) $L = 8$, (b) $L = 12$, (c) $L = 16$, and (d) $L = 24$.
**4.1.3.5 Arithmetic Complexity**

In this section we evaluate the computational complexity of the FC-FB scheme and compare it with the complexity of the polyphase filter bank structure. The focus here is on the FBMC/OQAM-type analysis filter bank, and we use the number of real multiplications and real additions per received complex symbol as the metric for the complexity. Since FFT and IFFT are the core modules in both types of filter banks, we start with their complexity. For a given transform length FFT and IFFT have the same complexity, so we talk only about the FFT complexity. When the transform length is a power of two, the split-radix algorithm is commonly considered to be the most efficient one [39] and the number of real multiplications and additions can be expressed as

\[
C_M = N(\log_2(N) - 3) + 4
\]

and

\[
C_A = 3N(\log_2(N) - 1) + 4,
\]

respectively. We consider also transform lengths of the form \(3\hat{N}\) where \(\hat{N}\) is a power of two. In these cases, three FFT’s of length \(\hat{N}\) are needed, together with \(\hat{N}\) radix-3 transforms.

![Figure 4-19: Examples of creating 2, 4, and 8 times wider subbands using the transition band shapes from SPR₂-optimized basic FC-FB designs with roll-off of 1, \(N = 1536\), and \(L = 12\). (a) RRC case. (b) RC case. (c) Arbitrary transition band case.](image_url)
Figure 4-20: Addition and multiplication rates for FBMC/OQAM type FC-FB’s, polyphase filter banks and basic OFDM processing for 512 subchannels out of which 300 are used. For FC-FB, the minimum overlap factor reaching the target SPR$_2$ is selected. For the polyphase FB’s, $K = 3$ and $K = 4$ are considered reaching SPR$_2$ of 50.0 dB and 62.7 dB, respectively.

A radix-3 transform takes 4 real multiplications and 12 real additions. Thus the resulting complexity is given by
\[ C_M = 3\hat{N}[\log_2(\hat{N}) - 5/3]) + 12 \]  
\[ C_A = 9\hat{N}[\log_2(\hat{N}) + 1/3]) + 12. \]

We assume that $M_{used}$ out of the $M = 2N/L$ subchannels of the FBMC/OQAM-type FC-FB structure are in use. Further we assume that generic complex FFT-domain weights are used. Real weights and certain trivial weight values could be utilized to reduce the complexity, but this is possible only if the earliest sampling phase is utilized. Additionally, the complex FFT-weights can also be used for subchannel equalization purposes. With these assumptions, a FC analysis filter bank implementation includes

- FFT of length $N$.
- $M_{used}(L-1)$ complex weight coefficients, each assumed to take 4 real multiplications and 2 additions.
- $M_{used}$ IFFTs of length $L$.

These calculations for one FFT block produces $M_{used}L_S/2$ complex symbols because the FC-processing produces two times oversampled output sequences.

Let us consider next the arithmetic complexity of a polyphase analysis bank with the same number of subbands, $M = 2N/L$, as in the FC-FB. This implementation includes

- FFT of length $M$.
- $KM$ real coefficients in the polyphase filter structure; the multiplication rate is double due to complex input.
- $2(K-1)M$ real adders.
- $M_{used}$ complex weights as single-tap subcarrier equalizers, each assumed to take 4 real multiplications and 2 additions.
The equalizer taps are implemented at the subcarrier symbol rate, otherwise the mentioned processing functions need to be implemented two times for each symbol interval to produce $M_{used}$ complex symbols.

Here single-tap subcarrier equalizer is included for both approaches. The complex FFT-domain weights of FC-FB can be used also for implementing multi-tap subcarrier equalizers with no additional complexity, whereas multi-tap subcarrier equalization would increase the complexity in the polyphase structure. However, such embedded channel equalizer design for FC-FB is not quite straightforward, and is beyond the scope of this paper. Therefore, this feature is not included in the comparisons of this paper.

Figure 4-20 gives a comparison of the two filter bank approaches in terms of arithmetic complexity in an example case with 5 MHz LTE like parameters [43]. Also the complexity of basic OFDM receiver processing is included for comparison. It is seen that the FC-FB has significantly lower complexity than the polyphase structure for the same SPR values. On the other hand, FC-FB has the complexity of about two times the complexity of OFDM. Naturally, OFDM has inferior spectral leakage performance compared to the other schemes.

4.1.4 FC-FB Parametrization Aspects for the 5 MHz LTE Case

We consider again the the 5 MHz LTE case introduced in Section 4.2.1.5. Here the focus is on the power spectral density of the transmitted signal in a 1 RB gap.

4.1.4.1 FBMC/OQAM Case with Commonality of Main Parameters

One basic approach for selecting the main parameters of the FC-FB scheme is to use the same sampling rate, the same subcarrier spacing, and the same parameters for the RBs as in the 5 MHz LTE system. We assume that FBMC/OQAM subchannels use the minimum feasible sampling rate, 2x symbol rate. Then the FFT length can determined as $N = ML/2$ where $L$ is the IFFT length and the subcarrier spacing is $L/2$ FFT bins. (For clarity of the discussion, we focus here on the analysis bank of the receiver.)

The choice of the FFT length, together with the overlap factor, defines the time interval of each processing block and has a significant effect on the performance of the scheme with fast-fading channels. However, this depends greatly on the used channel equalization structure, and the analysis of the fading sensitivity is left as a topic for future studies. On the other hand, increasing the FFT length gives better chances to shape the spectrum to improve spectral containment. In any case, we assume that the FFT length should be shorter than the RB length and eventually we are looking for a good trade-off between the spectrum control and fading sensitivity.

A third element in the tradeoff is implementation complexity. Increasing the overlap factor improves the spectral containment for a given IFFT length but it also increases the computational complexity as the ratio of the useful part of FFT to the FFT length is reduced.

The general results of Section 4.2.3 indicate that using IFFT length of 16 gives a reasonable compromise between the mentioned effects. This choice leads to the FFT length of 4096. Figure 4-21 shows a comparison of the spectrum leakage to the 12 subcarrier spectral gap in the 5 MHz LTE case with different overlap factors $1 - L_s/L$ for $L_s = 9$, $L_s = 12$, and $L_s = 16$. Since FC-FB is a periodically time-varying process, the spectrum
leakage varies with time within the FFT block. This figure shows the mean-squared distortion for each sample within the useful part of the IFFT block and for the 2nd to 11th subcarrier within the RB-wide gap. Since the FB subchannels have a roll-off of one, the first and 12th subcarrier are significantly overlapping with the active subcarriers at the edges, and are not included in the plots. In the experiment, it is assumed that all the 288 active subcarriers are transmitting offset-QPSK waveforms with equal power levels, and the interference level is relative to the active subcarrier power. We can see that the interference level is considerably higher during the first and last subcarrier symbols of the used part of the IFFT block. The average spectrum leakage levels over the output blocks are shown in Table 4-1. Figure 4-22 shows the inband interference in an active subcarrier due to the imperfections of the FC processing. Again, the interference dominates in the first and last samples of the IFFT block. All these results are obtained through a simulation model of the FC-FB filter bank system, and they are consistent with the results obtained through the analytical model.

![Figure 4-21: Interference level in 12 subchannels (1 RB) wide spectral gap for 512 subcarriers out of which 300 are in use for $N = 4096$ and $L = 16$. (a) Optimized design with $L_S = 9$. (b) RRC-design with $L_S = 12$. (c) RRC-design with $L_S = 14$.](image)
FBMC uses OQAM subcarrier modulation and the subcarrier signals are sampled at twice the OFDM symbol rate. The 1 ms sub-frame consists of 15 OQAM symbols or 30 subcarrier samples. Each 4096-point FFT is used to produce 9, 12, or 14 subcarrier samples with the overlap factors of 7/16, 4/16, and 2/16, respectively. Therefore, 3 or 4 FFT blocks are needed per sub-frame. However, in continuous transmission, the FFT blocks are not confined to RB or sub-frame boundaries. We can see that in the downlink transmission, the FBMC approach allows to increase the number transmitted bits per RB by the factor of 15/14 when compared to LTE with normal CP and by the factor of 15/12 when comparing to the extended CP variant.

However, since uplink users are not perfectly synchronized in time and frequency, guard bands need to be inserted between the RBs of different users and guard intervals are needed between different users’ transmission bursts exploiting the same subcarriers. Using just one subcarrier subband between different user’s groups of subcarriers, the users are well isolated even if they are not synchronized in time as long as the relative frequency offset is small compared with the subcarrier spacing of 15 kHz. One subcarrier guard band
can be implemented systematically, e.g., by leaving the upper edge subcarrier of each user’s transmission block to be unused.

To maintain spectrum control in the uplink, some subcarrier symbols in the beginning and at the end of the transmission burst must be set to zero and also the symbol pulses may have to be truncated. With the overlap factors of $7/16$, $4/16$, and $2/16$, it is possible to include 11.5, 12, or 14 OQAM symbols in each subcarrier of a sub-frame long transmission burst without the need for impulse response truncation. (Here 0.5 OQAM symbols mean just the real or imaginary part of the complex symbol, or effectively half the number of bits mapped to each complex symbol.) Depending on how well different uplink users are synchronized in time, additional guard intervals may be needed for the transition intervals. With this model of guard bands and guard intervals, the overheads in transmission capacity depend on the users data rates and also on the scheduling approach, as well as on the tolerable level of spectral leakage through the choice of the overlap factor.

Table 4-2 shows a comparison of the computational complexity of different multicarrier transmission schemes based on the number of multiplications and additions needed in the receiver analysis bank processing [32]. Also a single-tap subcarrier equalizer is included for each of the schemes. With the considered parameters, all the needed FFT and IFFT lengths are powers of two, and the complexity of the split-radix algorithm has been used as the basis. The possibilities of reducing the FFT complexity through FFT pruning is not considered here. The results are given for the case where all the 300 subcarriers are active, and also for the case where 60 subcarriers (5 RB’s) are active. We can see that the complexity of the FC-FB scheme is significantly lower than that of the polyphase FBMC implementation. The difference is pronounced when the number of active subcarriers is reduced. The complexity of the FC-FB with smaller overlap factor is about double the complexity of the basic OFDM scheme.

<table>
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**4.1.4.2 Multimode Parametrization Aspects**

Starting from the described FC-FB parametrization, it is possible to utilize the flexibility of the scheme on the following ways:

- Using several FBMC/OQAM multiplexes with different subchannel widths. With widened transition bands, it is possible to reach better spectrum control, e.g., to protect well nearby spectral slots which are sensitive to
interferences. On the other hand, widening the bandwidth while using the basic transition band shape would reduce the roll-off and, consequently, interferences between adjacent overlapping subcarriers with imperfect synchronization. This could be useful in various interference cancellation based schemes under consideration in EMPhAtiC.

- Using FMT multiplexes. Again, with widened subchannels, the roll-off can be reduced in the same way as mentioned above, leading to increased spectral efficiency.
- Using single carrier transmission. Basically, this can be considered as an FMT waveform with just one subcarrier. On the base-station side, this would be one element seen by the multi-mode FC-FB module, but especially in the mobile transmitter, simplified basic single-carrier processing would be compatible with the scheme.
- Different types or differently parametrized multiplexes could be processed simultaneously by a single FC-FB processing module as long as the constraints regarding the block sizes are satisfied. If differently parametrized FBMC/OQAM systems are not overlapping, their (near) orthogonality is maintained. As discussed in earlier chapters of this deliverable, it is possible to include subchannels with overlapping asymmetric transition bands, which can be used to glue together FBMC/OQAM multiplexes with different bandwidths without guardbands in between.

Using the earlier proposed parametrization with FFT length of $4096 = 2^{12}$, it follows that each IFFT length should be a power of two and also each subchannel bandwidth should be a power of two times the FFT bin spacing. This would severely restrict the parametrization possibilities of the mentioned transmission modes.

As another example case, we might consider using the basic IFFT length of 18. This would allow to use (the bold values are available also with 4096-point FFT):

- FBMC/OQAM subcarrier spacings of 15, 20, 26.67, 30, 40, 53.33, 60, 80 ... kHz
- FMT subchannel spacings of 30, 35, 41.67, 45, 55, 68, 33, 75, 95 kHz and the corresponding roll-offs of 1, 0.75, 0.5625, 0.5, 0.375, 0.2813, 0.25, 0.1875 ...

Using the LTE subcarrier spacing of 15 kHz in FC-implementation of FMT appears to be a difficult issue and needs further investigations.

### 4.1.5 Parametrization for the Demonstrator

Due to the fairly limited computational capabilities of the selected implementation platform, the 1.4 MHz LTE scheme is selected as parametrization for the demonstrator. The 1.4 MHz LTE uses $M = 128$ subcarriers, out of which 72 are active, such that 36 active subcarriers are allocated on both side of the unused DC-subcarrier. As for 5 MHz LTE, the 72 active subcarriers are scheduled in resource blocks of 12 subcarriers, i.e., there are altogether 6 resource blocks. In this case, the 0.5 ms resource blocks consist of 960 samples at the used sampling rate of 1.92 MHz.
4.1.5.1 FBMC/OQAM Case with Commonality of Main Parameters

The primary transmission mode is selected to be FBMC/OQAM. For the prototype implementation, power-of-two values are preferred as the transform lengths. Using \( L = 8 \) might be considered for practical implementation but it would lead to critical performance tradeoffs. \( L = 10 \) or \( L = 12 \) would be interesting choices if not limited to power of two’s whereas \( L = 16 \) is considered as a comfortable basis for the demonstrator (8 FFT bins per subcarrier spacing). According to discussion is Section 4.2.4.1, this choice leads to the long transform length \( N = 1024 \).

As for the feasible values for the non-overlapping block length, \( L_s = 10 \) provides good inband and out-of-band interference levels, as shown in Figure 4-23. Figure 4-24 shows the results also for \( L_s = 11 \) and \( L_s = 12 \) for comparison. The performance is tested in terms of inband interference in the FBMC/OQAM transmission link with ideal channel, as well as transmitted out-of-band spectrum leakage level beyond the last active subcarrier.

For \( L_s = 10 \), each 1024-point FFT is used to produce 5 OQAM symbols. For odd values of \( L_s \), the mapping of the OQAM symbol values to the short transform blocks would be more complicated, that is, different for even and odd data blocks. For example, with \( L_s = 11 \) a block would contain 5.5 OQAM symbols.

4.1.5.2 Weight Coefficients

While the optimal weight coefficients for odd \( L_s \) are real, for even values of \( L_s \) the optimal weight coefficients become complex. This is because half a sampling interval timing offset is introduced through the phase of the weight coefficients in order to reach nearly symmetric resampling. Figure 4-23 shows the performance with both optimal complex weight coefficients and suboptimal real weights (without the resampling time shift) for \( L_s = 10 \). The results show that, in this design case, differences in the inband performance are insignificant and the spectrum leakage increases by about 7...10 dB when the suboptimal real coefficients are used. When making the choice for the demonstrator, this loss in performance should be balanced with the increased implementation complexity.

Figure 4-23: Inband and out-of-band interference with the proposed configuration and weight coefficients, for both optimal complex FFT-domain weights and for suboptimal real weights.
The optimized weight coefficient magnitudes can be seen in Table 4-3 below. The weights are separately optimized for the suboptimal real case, but the differences can be considered marginal when compared to using the magnitudes of the complex weights. For complex weights, the phase factors are calculated as

$$\phi(k) = \exp(-j \frac{\pi \ell}{L})$$  \hfill (4.12)

where $\ell$ is the FFT bin index.

![Figure 4-18: Inband and out-of-band interference with reduced overlap factors, $L_s = 11$ (optimal real coefficient) and $L_s = 12$ (optimal complex coefficients).](image)

Table 4-3: Optimized weight coefficient magnitudes for $N = 1024$, $L = 16$, and $L_s = 10$

<table>
<thead>
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<th>$k$</th>
<th>Complex coefficients</th>
<th>Real coefficients</th>
<th>$k$</th>
<th>Complex coefficients</th>
<th>Real coefficients</th>
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<tr>
<td>4</td>
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<td>1.0000000000 1.000000000</td>
<td>12</td>
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<td></td>
</tr>
</tbody>
</table>
In the basic scheme, the set of real coefficients (suboptimal for even $L_e$, optimal for odd $L_e$) are the same for all subchannels. In case of optimal complex coefficients for even $L_e$, four different sets are needed for different FBMC/OQAM subchannels, i.e., different sets for subchannels $4k$, $4k+1$, $4k+2$, and $4k+3$, with $-90$, $-180$ and $-270$ degree phase rotations between them.

The coefficients are rather robust for quantization. Using just 10 bits for the fractional part causes only a minor effect on the performance.

4.1.5.3 Channel Equalization/Estimation and Synchronization Structures

The following two configurations are under consideration:

- Per-subcarrier processing after IFFTs in the receiver: The same algorithms as with polyphase FB’s would be used. Fixed FFT-domain weights are used, as described above.
- Embedding the channel equalization and synchronization offset compensation with the FFT-domain weights: Possibilities for improved performance and reduced complexity. Variable FFT-domain weights are used and they need to be updated after any changes in the channel estimates or synchronization parameters.

The benefits of embedded equalization and synchronization would be most significant on the uplink side or in ad-hoc networking, for accommodating simultaneous reception of asynchronous transmissions.

4.1.5.4 Options for Multimode Capabilities

The following options are considered for multimode configuration:

- FBMC/OQAM transmission with wider subcarrier bandwidths.
- FMT transmission, e.g., with 45 kHz symbol rate and 33% roll-off by grouping 3 basic subcarriers together. A 12 subcarrier resource block would contain 3 FMT subcarriers.
- Single-carrier transmission with selected bandwidth.

4.1.6 Comparison in Terms of Spectral Containment

Here we compare the average spectrum leakage level in different multicarrier systems, including basic CP-OFDM, enhanced OFDM, FBMC/OQAM with polyphase filter banks and FBMC/OQAM implemented using the FC-FB approach. The comparison is done with the parameters of the 5 MHz LTE case, looking at the spectrum leakage in a one resource block (12 subcarriers) wide gap. In OFDM systems, the CP length is chosen as 128 (quarter of the useful OFDM symbol duration) to facilitate large macro-cell operation and effective sidelobe cancellation in enhanced OFDM. This results in significant advantage for the FBMC/OQAM schemes in terms of spectrum and power efficiency.

The CP-OFDM case represents LTE without any additional methods for spectrum control. As the enhanced OFDM scheme we choose the combination of edge windowing and cancellation carriers, which turned out in Section 1.2 to be the most effective one, while still
exhibiting reasonable computational complexity. The polyphase filter banks utilize the PHYDYAS prototype filters with overlapping factors of $K = 3$ and $K = 4$ [23, 24]. In the FC-FB design, IFFT length of 16 and non-overlapping data blocklength of 10 is used.

Figure 4-25 shows a comparison of the selected schemes. The good spectral containment of FB schemes is clearly visible, as well as the nature of the tradeoffs of different FB schemes from the spectrum control point of view. Here the FC-FB system is optimized for minimum the spectral leakage in the closes subcarriers (SPR2 criterion), which creates flat spectrum in the gap.

![Figure 4-25: Average interface levels in 12 subcarriers wide spectrum gap in different multicarrier systems.](image)

### 4.2 Comparison in terms of implementation complexity

In this section the implementation issues based on MCFB and FCFC are considered in terms of complexity, memory and arithmetic usage. The analysis is done for one particular demonstrator specification and can be extended in the future for the rest of the LTE parameters.

#### 4.2.1 Demonstrator specifications

The LTE parameters for filter banks implementation are:

- Bandwidth 1.4 MHz;
- 1.92 MHz sampling rate;
- 128 subcarriers, 72 active.

The FC parameters are defined as:

- Short (I)DFT in 16 points;
- Long (I) DFT in 1024 points.
- 72 active subcarriers.
4.2.2 Realization of transmultiplexer system, synthesis bank

The block diagram of the synthesis OQAM filter bank is shown in Figure 4-1; the filters \( G_k(z) \) are derived from prototype filter with real taps, by complex modulation at \( M \) equidistant frequencies. The efficient realization is made of IDFT and polyphase filtering [46].

Figure 4-26: Transmultiplexer Synthesis Filter Bank

The filters \( G_k(z) \) are related to prototype filter \( p(m) \) as the following

\[
G_k(z) = P(z) W_k W_{M}^{L_p-1} = e^{\frac{2\pi k}{M}}
\]

where \( L_p \) is length of the prototype filter. Depending on the filter length, extra delay \( z^{-D} \) is introduced according to \( D = MP - L_p + 1 \).

The prototype filter can be decomposed to \( M \) polyphase components according to the following equation

\[
P(z) = \sum_{\lambda=0}^{M-1} z^{-\lambda} P_{\lambda}(z^{M})
\]

Based on the polyphase decomposition, the filter \( G_k(z) \) can be expressed using prototype components

\[
P_{\lambda} G_k(z) = W_{M}^{L_p-1} \sum_{\lambda=0}^{M-1} z^{-\lambda} W_{M}^{-\lambda k} P_{\lambda}(z^{M})
\]

The output signal from transmultiplexer, after some manipulations can be expressed as the following
\[ Y(z) = \sum_{\lambda=0}^{M-1} z^{-\lambda} P_{\lambda}(z^M) \sum_{k=0}^{M-1} W_M^{-k\lambda} W_M^{-k\lambda - 1/2} X_k \left( \frac{M}{z^2} \right) \]

\[ = \sum_{\lambda=0}^{M-1} z^{-\lambda} P_{\lambda}(z^M) Q_{\lambda} \left( \frac{M}{z^2} \right) \]

\[ = \sum_{\lambda=0}^{M-1} z^{-\lambda} S_{\lambda} \left( \frac{M}{z^2} \right) \]

\[ = \sum_{\lambda=0}^{M-1} z^{-\lambda} \left( S_{\lambda} \left( \frac{M}{z^2} \right) + z^{-1/2} S_M \left( \frac{M}{2z^2} \right) \right) \]

\[ \text{(4-16)} \]

The polyphase component \( S_{\lambda}(z) \) is convolution of the IDFT output bins and polyphase filter components.

\[ S_{\lambda}(z) = P_{\lambda}(z^2) Q_{\lambda}(z) \quad \text{(4-12)} \]

The term \( Q_{\lambda} \left( z^{M/2} \right) \) from (4-5) can be identified as IDFT of the vector constructed as product of the inputs and prototype dependant complex factors.

\[ Q_{\lambda} \left( \frac{M}{z^2} \right) = \sum_{k=0}^{M-1} W_M^{-k\lambda} W_M^{-k\lambda - 1/2} X_k \left( \frac{M}{z^2} \right) \]

\[ = \sum_{k=0}^{M-1} W_M^{-k\lambda} U_k \left( \frac{M}{z^2} \right) \quad \text{(4-18)} \]

Based on the (4-16), (4-17) and (4-18) the efficient polyphase realisation with IDFT is shown in Fig. 4-27 and it is discussed in detail in [38].
4.2.3 OQAM processor
The OQAM processor makes time-frequency grid, where every real symbol on the grid is surrounded by imaginary symbols and vice versa. This is performed using OQAM scheme, where time resolution is doubled. Fig. 4.28 shows basic OQAM processor for the synthesis bank.

![OQAM processor diagram]

Figure 4-28: OQAM processor, synthesis

4.2.4 Implementation details of synthesis bank
The implementation of the given LTE specifications is intended to be done on the FPGA type of a device. This implementation can be easily migrated to a signal processor.

Based on the defined sampling rate of 1.92 MHz, the working frequency is taken to be 64 times faster which gives 122.88 MHz, defining the processing rate of the group of 128 complex symbols every 8128 clock cycles.

The non-zero complex symbols at 1.92/128 MHz = 15 KHz are processed within OQAM processor, which generates the same number of real symbols at doubled rate of 30 KHz. The real symbols are multiplied with rotation factor $\theta_{n,k} = j^{k+n}$, where $k$ is index from frequency and $n$ index from time axis. The resulting outputs are upsampled twice and they appear at every 4096 clock cycles.

Further processing performs multiplications with complex time-frequency dependent $\beta_{k,n}$ complex constants. The obtained complex vector of 128 elements is then run throughout IDFT on every 4098 clock cycles, the time interval which determines the way of IDFT implementation in terms of resources like memories and multipliers.

The polyphase filtering is performed over 128 channels and output samples are upsampled by factor 64, which gives 1.92 MHz output sampling rate, and samples are changed on every 64 clock cycles at 122.88 MHz.

4.2.4.1 OQAM processing and
The OQAM processor, Fig. 4-29, takes input complex symbols and makes real $d_{k,n}$ symbols at doubled data rate. Finally, from real symbols it makes complex outputs by
the \( j^{k+n} \) rotation factor multiplication. The input group of 72 (number of active carriers) symbols feeds into the OQAM processor every 8192 clock cycles at 122.88 MHz. From the stored symbols, real symbol \( d_{k,n} \) is formed at double data rate from complex inputs by alternating real and imaginary part.

![Fig. 4-29: OQAM processor, Implementation](image)

The real symbols \( d_{k,n} \) are rotated by \( j^{k+n} \), forming complex inputs to the IDFT transform. The OQAM processor realisation is shown in Figure 4-29, where real symbol \( d_{k,n} \) generation and complex rotation is implemented using multiplexers according to timing diagrams shown in Figure 4-30.

![Fig. 4-30: OQAM processor, timing](image)

The formed complex symbols form OQAM processors are further rotated by the factor

\[
\beta_{n,k} = W_M^{\frac{L-1}{2}}
\]

which depend on the prototype filter length and subcarrier position.

**4.2.4.2 IFFT block**
The demonstrator filter bank IFFT block needs to be finished in 4094 clock cycles in order to meet the symbol rate. The number of the radix-2 butterflies is $N \log_2(N) = 896$, so there are around 4.5 clock cycles per butterfly to be performed. This implies simple IFFT architecture with shared butterfly for all calculations.

4.2.4.3 Polyphase Filtering

The output sample from polyphase filter can be represented as sum of the two polyphase components, from lower branches, $k < M/2$ and upper branches defined by $k + M/2$, i.e. $s_k(n) = y_k(n) + y_{k+\frac{M}{2}}(n)$.

The efficient realisation of the two polyphase outputs is shown in the figures bellow. The first $M/2$ polyphase components are calculated according to Figure 4-32, taking every second samples from the begging of delay line to convolution with appropriate polyphase component of the prototype filter.
The lower M/2 polyphase components are calculated by taking every second sample starting from the second position in delay line to convolution with appropriate polyphase component of the prototype filter.

\[ y_{k+\frac{M}{2}}(n) = \sum_{m=0}^{\frac{M}{2}-1} x_{k+\frac{M}{2}}(m) z^{-m} P_{k+\frac{M}{2}}(z^2) \]

Fig. 4-33: Second M/2 polyphase components.

Finally, the output component is sum of appropriate polyphase components from the first M/2 and the second M/2 branches.

The architecture for implementation of the polyphase filtering depend on the number of operations needed to be performed in defined time period of 4096 clock cycles. The overall number of multiply and accumulate is \(4 \times 128 = 512\), which enables to share single multiplier and accumulator for all calculations. The required memory space for polyphase delay line is \(128 \times 8 = 1024\) locations. Taking into account that 512 MAC operations should be done for 4096 cycles, the architecture with dual port memory for delay line and single MAC unit is chosen for polyphase filtering realisation, as shown in Figure 4-34.

The filtering is organised in the following way, the first sample to be calculated \(y_{k}(n)\), belongs to the lower polyphase branch, \(k < \frac{M}{2}\) (first M/2 branches), than the appropriate sample form upper branch \(y_{k+\frac{M}{2}}(n)\) is calculated. The output sample is sum of the appropriate samples from lower and upper branches.
The inputs $x_{nM}(k)$ are coming serially from IFFT block in natural order; they are stored into 128 consecutives locations using A memory addressing port. On the other hand, the outputs form polyphase lower $M/2$ and upper $M/2$ branches are calculated by applying proper addressing to memory port B and single MAC unit. Finally, the output sample is sum of the samples from lower and upper branches.

The lower part branches are addressed on the port B as the following,

$$Addr_B = Addr_A - i \times 256, \ i = 0,1,2,3.$$  

The shift of 256 locations corresponds to latency of $z^{-2}$ in the appropriate branch.

The upper part components are calculating addressing as the following,

$$Addr_B = Addr_A - 64 - i \times 256, \ i = 0,1,2,3.$$  

The shift of 64 corresponds to additional latency of $z^{-1}$ related to upper branches.

The coefficients of upper part are addressed as

$$Addr_{-M} = k + i \times 128, \ i = 0,1,2,3.$$  

The lower part coefficients are addressed as,

$$Addr_{-M} = k + 64 + i \times 128, \ i = 0,1,2,3.$$  

**4.2.4.4 Synthesis bank - implementation summary**

The overall resource usage for the synthesis filter banks is given by the consistent blocks:

- OQAM processor takes 1 block ram size of 128 locations;
- Phase shifter, one multiply-accumulate unit;
- IDFT takes 4 block ram memories, size of 128 locations and one complex butterfly which takes 3 multiplier and 6 adders;
• Polyphase filter takes 2 block RAM memory size of 1024 locations, one ROM for coefficients storage, and two multiply-accumulate unit, for realisation real and imaginary branch of polyphase filters.

4.2.5 Realization of transmultiplexer system, analysis filter bank

The task analysis filter bank is to reconstruct sent symbols over synthesis filter bank. The basic realisation of analysis filter bank is shown in Fig. 4-35. Based on the fact that filters are derived from prototype by equidistant frequency shift, the implementation can be performed in efficient way using DFT and polyphase filtering, [14],[46] and [38].

The filters $F_k(z)$ are derived from prototype filter $p(m)$ by complex modulating and time reversing as the following

$$f_k(m) = g^*(Lp - 1 - m) = q(m)e^{2\pi i \left(m - \frac{Lp - 1}{2}\right)}, m = 0,\ldots, L_p - 1$$

$$q(m) = p(L_p - 1 - m)$$

$$G_k(z) = Q(zW_M^k)W_M^{L_p-1}$$

The received sub-channel $\hat{X}_n(z)$ can be expressed

$$R(z)G_k(z) = W_M^{L_p-1} \sum_{\lambda=0}^{M-1} z^{-(M-\lambda-1)}R(z)Q_{M-\lambda} \left( z^M W_M^{(\lambda+1)k} \right)$$

$$= W_M^{L_p+1} \sum_{\lambda=0}^{M-1} z^{-(M-\lambda-1)}R(z)Q_{M-\lambda} \left( z^M W_M^{\lambda k} \right)$$

These equations lead to the following implementation based on polyphase filtering and DFT.

![Fig. 4-35: Analysis filter bank.](image)
4.2.6 Implementation details of analysis filter bank

The working frequency is considered to be 64 times greater than sampling frequency, which gives 122.88 Mhz. Basically, there are 64 clock cycles per input samples at 1.96 MHz, which allow for sharing of resources as much as possible.

The first stage in the analysis filter bank is signal block-partitioning into polyphase components. There are M components derived through decimation by M/2. After signal ‘blocking’, the polyphase filtering is applied on M channels.

The filtered polyphase components form a vector of 128 elements. This vector is then passed through DFT, which is resulting in output vector that needs to be multiplied by time-frequency defined constants $\beta_{n,k}$. The output vector $\tilde{X}_{n,k}$ is processed with inverse OQAM processor in order to obtain estimated symbols.

4.2.6.1 Signal blocking and Polyphase filtering

The received complex signal sampled at 1.92 MHz is blocked into 128 polyphase component; using property that decimation rate is 64, the relation between first half and second half of the components can be represented as in Fig. 4-37.
The signal blocking is implemented using two linear dual port memories with 1024 locations. The inputs are available on every 64 system clocks, which corresponds to write enable to memory. Every 128 samples, e.g. every $64 \times 64 = 4096$ clock cycles new block is ready for polyphase filtering, that’s why there are $4096/(128 \times 4) = 8$ cycles per multiply-accumulate operation for real and imaginary part of filtering. Using the fact that the number of cycles is greater than 1 per operation leads to simple architecture with one MAC unit being able perform all the filtering operations.

The addressing is similar as described in the section related to synthesis bank polyphase filtering. The complex-valued results from MAC unit are stored into intermediate dual port RAM, which feeds the DFT block at every 4096 clock cycles (64 input samples).
4.2.6.2 **DFT block**

The same discussion applies for the DFT realisation as was presented in sub-section related to the IDFT for synthesis filter bank. The single radix-2 butterfly can be shared for all DFT calculation.

4.2.6.3 **\( \beta_{n,k} \), filter length dependent constants, analysis bank**

The filter length constant are introducing phase shift over DFT outputs as \( W_{\frac{Lp+1}{2}} \), which can be implemented using one multiplier.

4.2.6.4 **OQAM processing, analysis filter bank**

The analysis filter bank OQAM processing performs reverse operation w.r.t. the one at synthesis side, as can be seen in Figure.

\[
\begin{align*}
\hat{\beta}_{n,k} & \\
\theta'_{n,k} & \\
\text{Re}\{ \} & \\
\downarrow \quad 2 & \\
\downarrow \quad 2 & \\
\text{Im}\{ \} & \\
\end{align*}
\]

Fig. 4-39: OQAM processor, analysis filter-bank.

The implementation uses two dual port memories with 128 locations, for storing delayed symbols and the current complex-valued symbol.

\[
\begin{align*}
\text{Re}\{ \} & \\
\downarrow \quad 2 & \\
\downarrow \quad 2 & \\
\text{Im}\{ \} & \\
\end{align*}
\]

Fig. 4-40: OQAM realisation.
4.2.6.5 **Analysis filter bank design summary**

The overall resource usage for analysis bank is:

- Polyphase filter takes 4 block RAM memories of 1024 locations for data storage, and one 1024 ROM for the filter coefficients and 2 MAC units;
- DFT 128 takes 4 block RAM memories of 128 locations, and one complex butterfly which takes 3 multiplier and 6 adders;
- Phase shifter can be implemented using one MAC unit;
- OQAM analysis takes 2 BRAM memories with the size of 128 locations.

4.2.7 **Realisation of fast convolution filter banks (FCFB), synthesis bank**

Fast convolution concept is given in the first part of section 4; also the realisation based on fast convolution is covered in [12.13]. The synthesis and analysis filter bank are calculated using approximations in the frequency domain and signal overlapping in the time domain.

4.2.7.1 **FC synthesis filter bank**

Fig. 4-41 shows two adjacent channels in the fast convolution realisation of the synthesis filter bank. The number of active carriers (72) corresponds to number of channels, which are processed with short DFT in 16 points.

![Fig. 4-41: Synthesis FCFB.](image-url)
The FCFB can be divided into several groups according to processing elements:

- Short overlap for 72 input channels;
- Phase shifting;
- Short DFT 16 for all input channels;
- Masking, frequency shaping with $W_k, k = 0, \ldots, 15$.
- Forming samples for long IFFT by adding two adjacent channels;
- Output signal extracting.

The channel spacing is 8 FFT bins of 1024 points, which in frequency gives $8 \times 1.92 MHz / 1024 = 15 KHz$ spacing of the adjacent channels. There are 16 coefficients per channel which overlap with two adjacent ones, as shown in Fig. 4-41.

**Fig. 4-41: Adjacent channels frequency overlap.**

### 4.2.7.2 Short overlap implementation

Fig. 4-42 illustrates short overlapping in time. The input symbols are coming in burst of 72 complex numbers at every 4096 clock cycles. In order to properly perform overlapping, 16+No samples have to be stored for every channel, which gives memory size of 2*1296 locations for real and imaginary samples storage.

**Fig. 4-12: Short overlapping.**
The easiest way to implement overlapping is to use 2 dual port memories, for real and imaginary part each. The addressing port A is used for input samples storage and port B for addressing samples that are fed to DFT 16.

4.2.7.3 Short DFT 16
The short, 16-point DFT (DFT 16) should be applied over 72 active carriers. The available time for all DFTs to be finished is $16 \times 4096/2$ clock cycles. The time for one DFT is around 455 cycles. This defines DFT 16 implementation architecture which consists of 64 complex butterflies plus 16 read in and 16 read out cycles.

Comparing available and required clock cycles, the architecture with one shared butterfly can be implemented, which requires one real multiplier.

4.2.7.4 Masking, frequency domain filtering and phase shift
This block can be implemented using only one multiplier, because DFT 16 outputs are coming out serially. The phase shifting requires one complex multiplication, which can be made of 3 real multipliers. Because there is more than 3 cycles per multiplication, $4096/16/72 = 3.5$, one shared multiplier can perform phase shifting.

4.2.7.5 IFFT data forming
The IFFT formatting block has a task to form 1024 IFFT bins, to be processed. The adjacent channels overlaps in 8 frequency bins; therefore current incoming bins need to be added to previously generated part, and meanwhile the IFFT processor needs to operate over 1024 already prepared bins.

The logical choice is to use two dual port memories to form so called Ping-Pong structure, which enables processing over 1024 bins, while new 1024 bins are being prepared.

4.2.7.6 IFFT block
The IFFT block has $16 \times 4096$ clock cycles to perform 1024 IFFT, which gives 6.4 clock per butterfly; clearly the shared architecture can be applied which can be implemented with one multiply-accumulate unit.

4.2.7.7 Output signal extractor
The samples at the output are taken from IFFT outputs as follow:
• From the first IFFT 1024-Lo samples are taken;
• From the following IFFTs, except the last one, Lo:1024-Lo samples form the output;
• From the last IFFT in the frame.

The output buffer is made of two dual port RAMs with 1024 locations for real and imaginary part of the baseband signal at 1.92 MHz, with 64 clock cycles per sample.

**4.2.7.8 FC synthesis bank overall resource usage**

The overall resource usage for FC synthesis is:

• Short overlap takes 2 block RAM memories of 1296 locations;
• Short DFT 16 takes one complex butterfly, which consumes 3 multipliers and 6 adders;
• Frequency domain filtering and phase shift takes 2 multipliers and 2 adders;
• IFFT bin preparing unit takes 4 BRAM memories of 1024 locations;
• IFFT takes 3 multipliers, 6 adders, 4 BRAMs of 1024 locations for processing and 1 ROM for twiddle factors storage;
• Output signal extractor just needs to read out from IFFT BRAM where final results are stored.

**4.2.8 FC analysis filter bank**

The analysis filter bank can be divided into the following stages:

• Input block overlap;
• Long DFT, in 1024 points;
• Bins extracting;
• Masking, or frequency-domain filtering;
• IFFT, i.e. getting signal back in the time domain;
• Phase shifting;
• Output signal extracting.

**4.2.8.1 Input block overlap**

The input baseband complex signal is sampled at 1.92 MHz; the system clock is suggested to be 64 times the sampling clock, which results in operation clock at 122.88 MHz.
Fig. 4-44: Overlapping in the synthesis bank.

Fig. 4-44 illustrates overlap processing. The current block to be processed with FFT 1024, consist of $L_o = 128$ samples from previous block and $N_{\text{fft}} - L_o = 896$ samples from the current block. The signal overlap can be implemented using 2 dual port block RAM memories, each of $128 + 1024 = 1152$ locations. The output of these two memories, in burst-mode feeds serially the DFT processor.

**4.2.8.2 Long DFT**
The DFT calculation has to have $64 \times 1024$ clocks available to be performed. This implies realisation with only one shared butterfly. As has been mentioned earlier, it takes 3 multipliers and 6 adders, 2 dual ports block RAM to store inputs and for intermediate storages, and 2 dual ports block RAM for output storage.

**4.2.8.3 Bin extracting and filtering**
The bin extracting is actually reading process, form DFT output memories, of desired 16 consecutive frequency bins in order to be filtered in the frequency domain by weighting. Directly after multiplying by $W_k$, signal is serially fed into the IFFT 16 block.

**4.2.8.4 Short IFFT and phase shifting**
The short IFFT transforms signal form frequency to time domain, from 72 defined channels extracted and filtered from long DFT. The time required for channel transformation is $64 \times 1024/72/2 = 455$ clock cycles per IFFT 16, which implies usage of the one shared IFFT 16 unit.

The phase shifting requires one complex multiplication, but with enough clock cycles, it can be made of one shared multiplier.

After phase shifting, 16 samples of each 72 channels are stored into dual port block RAM, with $2 \times 1152$ locations.

**4.2.8.5 Output signal extractor**
The samples at the output are taken from FFT outputs channels according to the following:

- From the first FFT 16-No samples are taken, the first No are neglected;
• From following FFTs, except the last one, No:16-No samples forms the output;
• From the last one FFT in the frame No: 16 samples are taken.

4.2.8.6 Analysis FC-FB overall resource usage
The overall resource usage for FC analysis bank is given as:

• Long overlap takes 2 BRAM sizes of 1152 locations;
• Long DFT 1024 takes 3 multipliers, 6 adders, 4 BRAMs of 1024 locations for processing and 1 ROM for twiddle factors storage;
• Frequency domain filtering takes 1 multiplier and 1 adder;
• IDFT 16 takes 3 multipliers and 6 adders, and one BRAM size of 1152 locations for the result storage;
• Phase shifting, takes 1 multiplier and 1 adder;
• Output signal extractor requires just proper reading form IDFT16 result memory.

4.2.9 Conclusion related to implementation
For the given LTE parameters, the implementation is very simply, because of low sampling rate of 1.92 MHz, and there are a large amount of the clock cycles per sample which enables resource sharing and engagement quite small number of MAC units and memories.

Comparison in the term of resource usage for synthesis bank implemented using polyphase filter banks and fast convolution implementation is given in Table 4-1.

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<th>FB (filter banks)</th>
<th>FC (fast convolution)</th>
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<tr>
<td></td>
<td>2 BRAM 1024</td>
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</tr>
<tr>
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The complexity of the both designs, for given specifications, are slightly different and filter bank realisation consumes less resources, but this can be changed for specifications for wider bandwidth when other components are included, like equalizer and synchronisation.
The analysis bank has the similar results in the term of resource usage and can be seen in Table 4-2.

Table 4-2 analysis, FB and FC

<table>
<thead>
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<th>Analysis</th>
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<th>FC (fast convolution)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multipliers</td>
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The overall resource usage for the basic synthesis-analysis filter banks, realised using polyphase filter and fast convolution are very similar, with small advantage to polyphase filtering structure. This advantage can be even neutralized for other specifications, and when all processing elements are included, like synchronisation, channel estimator and equalizer.

The analysis related to the named processing block will be included in the future work related to implementation issues.

For the LTE with larger bandwidth the realisation could be more demanding in the terms of resources. This analysis for all possible LTE specifications will be derived later on during the project.
5. Prototype pulse optimization for FB-MC modulations

5.1 Introduction

It is well known that propagation conditions at the 400MHz band are much more benign than propagation conditions at higher carrier frequencies, such as those used for the LTE system. This fact originates larger delay spreads in the time domain of the observable channel, and it is for this reason that channel frequency selectivity becomes an important issue in the deployment of PMR systems. This is especially true in FB-MC modulation systems, which are well known to be quite sensitive to the effect of severe channel frequency selectivity. In this section we will see how to partially compensate this effect in the waveform design of the filterbank system, by explicitly taking into account the channel frequency selectivity in the filterbank prototype design.

5.1.1 Effect of channel frequency selectivity in FB-MC modulations.

The main idea behind filter-bank multicarrier modulations stems from dividing the transmission bandwidth into small subbands, so that the channel response observed at each subband is approximately flat in the frequency domain. When this is the case, one can implement the channel equalizer by simply multiplying the received signal by a single tap at each subcarrier. If the number of carriers is sufficiently high and the channel has relatively smooth variations over the transmission bandwidth, this simple equalizer is able to provide a very accurate reconstruction of the transmitted signal.

In practice, however, the use of a high number of subcarriers leads to a significant increase of the computational complexity of the transceiver, as well as a higher transmission latency. These two effects strongly limit the number of available subcarriers that can be deployed in practice, regardless of how severe the channel frequency selectivity might be. In these circumstances, the channel frequency selectivity is observed as some residual inter-symbol and inter-carrier interference terms, which cannot possibly be compensated by the use of single-tap per-subcarrier equalizers. A way to mitigate the effect of this distortion is by the use of more complex equalizers at the receiver, see e.g. [46] and references therein. However, this also comes at the expense of higher computational complexity, since in practice the system needs a different equalizer to compensate each subcarrier link.

In this section, we will explore an alternative way of mitigating the effect of high channel frequency selectivity without increasing either the computational complexity or the global latency of the system. The objective is to minimize the residual distortion in the filterbank prototype design. To that effect, it is crucial to fully characterize the channel frequency selectivity effect on this residual inter-symbol and inter-carrier interference.

Let us consider a FB-MC/OQAM system with an even number of subcarriers (denoted by \( M \)) and critically sampled, uniform and exponentially modulated filterbanks. The transmit filterbank is constructed by exponentially modulating a prototype pulse \( p[n] \), of length \( MK \), where \( K \) will be referred to as the overlapping factor. We will assume for simplicity that the receiver filterbank is also constructed from the same prototype, namely \( p[n] \). This prototype pulse can be designed so that the composite filterbank achieves perfect reconstruction under ideal channel conditions. We can synthesize the perfect reconstruction (PR) conditions as follows. Let \( P \) denote a \( M \times K \) matrix that contains, at
the $k$th row, the $k$th Type-I polyphase component of $p[n]$. Next, consider a $M \times (2K-1)$ matrix $R(p, p)$ defined as

$$R(p, p) = P \ast (J_M P)$$

where $J_M$ is the $M \times M$ anti-identity matrix and $\ast$ denotes row-wise convolution between matrices\(^5\). Using this definition, the orthogonality conditions that guarantee perfect reconstruction (PR) of the composite filter-bank can be expressed in compact form as

$$\left( I_2 \otimes (I_{M/2} + J_{M/2}) \right) R(p, p) = \begin{bmatrix} 0_{M \times (K-1)} & 1 & 0_{M \times (K-1)} \end{bmatrix}$$

where $0_{M \times (K-1)}$ is an all-zero matrix of dimensions $M \times (K-1)$ and $1$ is an all-ones column vector (in fact, the position of the column vector $1$ within the matrix on the right hand side may be different from the one above). When the prototype pulse $p[n]$ presents either symmetry or anti-symmetry in the time domain, one can see that the last $\frac{M}{2}$ equations in (5.1) become superfluous and can be obviated.

Now, as mentioned above, the conditions in (5.1) can only guarantee perfect reconstruction under ideal channel conditions. In the presence of channel frequency selectivity, the receiver will observe a residual inter-symbol and inter-carrier interference at its output. Under some technical conditions, it is possible to obtain a very good approximation of the residual distortion power at the output of the receiver. These conditions can be summarized as follows:

- The prototype pulse $p[n]$ is obtained by discretisation of a real-valued differentiable analog waveform $p(t)$, so that
  $$p[n] = p\left( n - \frac{MK + 1}{2} \frac{T_s}{M} \right), \quad n = 1, \ldots, MK$$
  where $T_s$ is the sampling period. We will also assume that the prototype pulse $p$ is either symmetric or anti-symmetric in the time domain, and that $p(t) = 0$ when $t = \pm KT_s/2$ (i.e. at the edges of its support).

- The complex symbols are drawn from a bounded constellation, and their real and imaginary parts are independent, identically distributed random variables with zero mean and power $P_s/2$.

- The channel has a finite impulse response.

Under the above conditions, we can establish an approximation to the residual distortion power at the output of the receiver that is valid under the assumption of a large number of subcarriers. To establish these results, we need some additional definitions. Let us denote by $p$ and $q$ two generic prototype pulses, of length $MK$. Let $P$ and $Q$ denote two $M \times K$

\[^5\text{That is, for two matrices } A \text{ and } B \text{ of appropriate dimensions, } A \ast B \text{ is a matrix whose } k\text{-th row is the convolution between the } k\text{-th row of } A \text{ and the } k\text{-th row of } B.]
matrices that contain, at each of their rows, the polyphase components of the pulses $p$ and $q$ respectively. We define the two matrices
\[
R(p,q) = P \ast (J_M Q) \\
S(p,q) = (J_2 \otimes I_{M/2}) P \ast (J_M Q)
\]
where $\otimes$ denotes Kronecker product. These two matrices will be very useful to characterize the asymptotic distortion of the FB-MC system in what follows.

5.1.1.1 Residual distortion caused by channel frequency selectivity under PR conditions

Let us first analyze the case where the pulse achieves PR under frequency flat channels. Let us denote by $H(\omega)$ the channel frequency response, and by $H'(\omega)$ its first order derivative. We assume that a single tap per-subcarrier equalizer is implemented at the output of the receive filterbank. This equalizer simply multiplies the symbols received at the $k$th subcarrier by $H'(\omega_k)$. We have been able to show that, under PR conditions and up to an error of order $O(M^{-4})$, the residual distortion power observed at the $k$th subcarrier (corresponding to the frequency $\omega_k$) is given by the expression [47]

\[
P_e(\omega_k) = \frac{2P}{M^2} \left| \frac{H'(\omega_k)}{H(\omega_k)} \right|^2 \eta(p)
\]

where $\eta(p)$ is defined as

\[
\eta(p) = \frac{1}{M} \left[ R(p,p') R^T(p,p') U^+ + S(p,p') S^T(p,p') U^- \right]
\]

and where $U^+ = I_2 \otimes (I_{M/2} + J_{M/2})$ and $U^- = I_2 \otimes (I_{M/2} - J_{M/2})$. In the above equations, $p'[n]$ denotes sampled version of the derivative of the prototype pulse in (5.2).

Some comments are in order. First of all, observe that the residual distortion power depends critically on the quotient between the derivative of the channel frequency response and the response itself at the central frequency of the sub-band. Hence, the distortion power does not decrease with increasing signal power (or input SNR), and depends on the degree of variation of the real and imaginary parts of the channel frequency response. In this sense, it is useful to point out that a timing error at the receiver in practice manifests itself as a linear phase distortion of the channel frequency response $H(\omega)$. Hence, even if the transmission channel presents an ideal frequency response, timing errors will lead to a residual distortion with power approximately given by (5.3).

On the other hand, it is important to point out that the only influence of the prototype pulse $p[n]$ on the residual distortion power is through the quantity $\eta(p)$. In other words, the effect of the channel frequency response $H(\omega)$ and the effect of the prototype pulse $p[n]$ are completely decoupled in the residual error power $P_e(k)$. Hence, one can consider the minimization of (5.4) as a function of the prototype pulse samples in order to derive prototype pulses that are robust against the effect of timing errors and strong channel frequency selectivity. This is precisely the type of approach that we take in Section 5.3.
5.1.1.2 Residual distortion caused by channel frequency selectivity under nearly PR conditions.

We will see in the following section that restricting our attention to pulses that have the PR property might not always be the best design option. In fact, it is sometimes reasonable to consider pulses for which the reconstruction conditions in (5.1) hold only approximately. Indeed, if the residual distortion introduced by the pulse is below the distortion introduced by the channel frequency selectivity, it will go unnoticed in terms of system performance. Furthermore, the transceiver can always tolerate a certain amount of total distortion, as long as its power stays well below the operative background noise power. For all these reasons, it is interesting to investigate the residual distortion of a FB-MC system caused when the pulse is only nearly PR (NPR) in the presence of channel frequency selectivity.

As a result of the work carried out in WP2, we have been able to show that -using the same techniques as in [47]- one can approximate the distortion at the output of the single tap per-subcarrier equalizer corresponding to the subcarrier $\omega_k$ as

$$P_e(\omega_k) = \frac{P}{M} \left[ (U^R(p,p) - I)(U^R(p,p) - I)^T \right] + \text{Im} \left[ \frac{H'(\omega_k)}{H(\omega_k)} \right] \frac{4P}{M^2} \left[(U^R(p,p) - I)R^T(p,p') \right]$$

$$- Re \left[ \frac{H''(\omega_k)}{H(\omega_k)} \right] \frac{2P}{M^3} \left[(U^R(p,p) - I)R^T(p,p'') \right]$$

$$+ \frac{H'(\omega_k)}{H(\omega_k)} \frac{2P}{M^2} \left[R(p,p')R^T(p,p') + S(p,p')S^T(p,p')U^+ \right.\left. + S(p,p')S^T(p,p')U^- \right]$$

where $H(\omega)$ is the channel frequency response, $H'(\omega), H''(\omega)$ denote its first and second order derivatives and where the matrices $R(p,q), S(p,q), U^+$ and $U^-$ are as defined above. Finally, $I$ denotes the matrix on the right hand side of (5.1) and $p[n]$ and $p'[n]$ denote the sampled versions of the first and second order derivatives of the prototype pulse in (5.2) respectively. This result becomes more exact as the number of subcarriers $M$ increases, and the approximation is valid up to an error of order $O(M^{-3})$.

It can readily be seen that this formula provides a very good approximation of the residual distortion power, even for relatively low values of the number of subcarriers. On the other hand, it should be pointed out that if the prototype pulse $p$ is such that the perfect reconstruction conditions in (5.1) hold, and therefore $U^R(p,p) = I$, the above formula becomes equal to the one in (5.3), as one would have expected.

Now, it is clearly observed from the expression of $P_e(\omega_k)$ that the system distortion will critically depend on the channel response that is observed at each subcarrier. For this reason, it seems reasonable to consider the total distortion observed averaged across the total spectrum, which can be expressed as
\[ P_e = \frac{P}{M} \left[ (U^T R(p,p) - I)(U^T R(p,p) - I)^T \right] + \alpha \frac{4P}{M^2} \left[ (U^T R(p,p) - I)R^T(p,p') \right] - \beta \frac{2P}{M^3} \left[ (U^T R(p,p) - I)R^T(p,p') \right] + \gamma \frac{2P}{M^3} \left[ R(p,p')R^T(p,p') \right] + S(p,p')S^T(p,p')U^T, \]

where the coefficients \( \alpha, \beta, \) and \( \gamma \) are defined as

\[
\alpha = \frac{1}{2\pi} \int_0^{2\pi} \text{Im} \left[ \frac{H'(\omega)}{H(\omega)} \right] d\omega, \quad \beta = \frac{1}{2\pi} \int_0^{2\pi} \text{Re} \left[ \frac{H'(\omega)}{H(\omega)} \right] d\omega, \quad \gamma = \frac{1}{2\pi} \int_0^{2\pi} \left| \frac{H'(\omega)}{H(\omega)} \right|^2 d\omega.
\]

In practice, it may be difficult to compute \( \alpha, \beta, \) and \( \gamma \) from the channel response, since these parameters need to be obtained using numerical integration in \( H(\omega), \ H'(\omega) \) and \( H''(\omega). \) We can avoid the computation of these integrals using classical complex integration techniques and the Cauchy residue theorem. More specifically, let \( H(z) \) denote (with some abuse of notation) the zeta transform of the channel impulse response, which is assumed to be of finite length \( L_h + 1. \) Let \( z_1, \ldots, z_{L_h} \) denote the zeros of this transform on the complex plane, and assume that they are all different and that there are no zeros on the unit circle. Let \( I_1 \) (respectively \( I_2 \)) denote the set of indexes of the zeros that are located inside the unit circle (respectively outside the unit circle), so that \( |1, \ldots, L_h| = I_1 \cup I_2, \ I_1 \cap I_2 = \emptyset. \) Then, we can express \( \alpha, \beta, \) and \( \gamma \) as functions of these zeros as follows

\[
\alpha = -|I_2|, \quad \beta = -|I_1| + 2\pi \sum_{l \in I_1, m \in I_2} \frac{z_l}{z_l - z_m},
\]

\[
\gamma = \sum_{l \in I_1, m \in I_1} \frac{z_l z_m^*}{1 - z_l z_m^*} + \sum_{l \in I_2, m \in I_2} \frac{z_l z_m^*}{z_l z_m^* - 1}.
\]

The roots of \( H(z) \) are easily obtained from e.g. the channel impulse response, using conventional polynomial rooting techniques. These techniques turn out to be much more accurate and efficient than the computation of numerical integration.

In Section 5.4 we will propose a procedure to optimize the pulse coefficients in order to optimize \( P_e \) according to the value of the parameters \( \alpha, \beta, \gamma, \) which can be inferred from the channel frequency response. We will need to assume that both transmitter and receiver have access to channel state information, and they agree on the type of pulse to be used accordingly. On the other hand, one could further simplify the pulse design procedure by further averaging the expression of the total distortion \( P_e \) with respect to different channel realizations. This would significantly relax the necessity of having access to instantaneous channel information at the transmitter side, thus simplifying the optimization and updating procedure.
5.2 Traditional approaches in prototype pulse design

The design of prototype pulses for filterbank transceivers is nowadays a rather mature research problem (see [48] for a recent extensive overview on the subject). Two different aspects related to prototype pulse design have received a lot of attention in the literature: the pulse structure, and the optimization criterion. Let us separately discuss these two aspects of the design problem in what follows.

5.2.1 Pulse structure

Traditionally, there have been two different ways of constructing the prototype pulses for FB-MC communications. The first approach assumes that the pulses are discretised versions of certain analog waveforms, which have certain good properties in the frequency domain. The most important examples of this type of pulses are the Extended Gaussian Functions (EGF) [49], the Optimal Finite Duration Pulses (OFDP) [50], pulses derived from the Isotropic Orthogonal Transform Algorithm (IOTA) [17], or pulses derived using the frequency sampling technique [30]. In all these cases, the prototype pulse will only have nearly perfect reconstruction (NPR) conditions, and consequently some remaining distortion will always be present at the output of the receive filterbank, even under frequency flat channels. If the analog waveforms are sufficiently well behaved, this remaining distortion will be lower than the background noise present in the system, and therefore no degradation will be observed in practice.

It is worth pointing out that the selection of a particular analog waveform from which our prototype filter is synthesised does not rule out the possibility of optimizing the pulse shape in order to meet a certain optimality criterion. This is because in practice the analog waveform will be parameterised with respect to a small number of variables that can be tuned in order to optimize the pulse response. We will see this in more detail in Section 5.4 below.

The second traditional approach for designing the prototype pulse structure directly optimizes the samples of the pulse in the digital domain [51]. The advantage of this is that the number of free parameters is much larger than in the analog sampling approach. Typically, some of these degrees of freedom are devoted to guaranteeing that the perfect reconstruction conditions hold, see e.g. [52]. This effectively reduces the total number of degrees of freedom, which simplifies the underlying optimization procedure. However, the number of remaining degrees of freedom is still large, and this makes it difficult to find numerically stable procedures that are able to converge to the optimum solutions, especially when the number of subcarriers is relatively large. Furthermore, as pointed out above, the PR conditions may be too strict in communication scenarios with some additive background noise, where a certain degree of pulse reconstruction distortion may well be allowed. We will further explore this approach in Section 5.3 below.

5.2.2 Optimization criterion

Regardless of which prototype pulse structure is selected, there typically exist some remaining variables that can be optimized according to a certain optimization criterion. These variables are either analog waveform parameters (first approach above) or generic
degrees of freedom in terms of pulse samples after forcing the PR constraints (second approach).

It is typically desired that the prototype pulse achieves high frequency selectivity together with the highest possible stopband attenuation. It is for this reason that the most common design criterion has been the minimization of the stopband energy of the pulse. More specifically, if $P(\exp(j\omega))$ denotes the frequency response of the prototype pulse, one could design the pulse coefficients so that the following cost function is minimized

$$\eta_{\text{OUT}}(p) = \int_{-\infty}^{\infty} |P(\exp(j\omega))|^2 d\omega$$

(5.5)

where $\omega_s$ is the stopband frequency of the pulse, typically fixed to $\omega_s = \frac{3\pi}{2M} (1 + \delta)$ where $\delta$ is a certain roll-off factor. This approach is followed in e.g. [51][49][14][53]. It can be shown that the above cost function $\eta_{\text{OUT}}(p)$ can be expressed as a certain quadratic form, namely $\eta_{\text{OUT}}(p) = p^T Q p$, where $p$ contains the prototype pulse samples and where $Q$ is a certain eigenfunction-based matrix (see [51] for further details). There exist in the literature some other variations on this criterion, such as the minimax criterion or the peak-constrained least squares criterion, which try to minimize some quantities directly related to the energy of the stopband (see [54] for further details).

A famous alternative for prototype pulse optimization seeks to minimize the product of localization measures in the time and frequency domains. In this case, the cost function is chosen as [55] $\eta_{\text{TF}}(p) = (4m_2M_2)^{-1/2}$, where $m_2$ and $M_2$ are the second order moments in the time and frequency domain, given by

$$m_2 = \frac{1}{||p||^2} \sum_{k=-\infty}^{\infty} \left( k - \frac{1}{2} - T \right) \left( \frac{p[k] + p[k-1]}{2} \right)^2, \quad M_2 = \frac{1}{||p||^2} \sum_{k=-\infty}^{\infty} \left( p[k] - p[k-1] \right)^2$$

and where $T$ is the time gravity center given by

$$T = \frac{\sum_{k=-\infty}^{\infty} \left( k - 1/2 \right) \left( p[k] + p[k-1] \right)^2}{\sum_{k=-\infty}^{\infty} \left( p[k] + p[k-1] \right)^2}.$$ 

This optimization criterion is very suitable to high mobility scenarios where it becomes imperative to have well concentrated pulses in both time and frequency domains.

As part of the work in WP2 of the Emphatic project, we have explored an alternative pulse optimization criterion based on the minimization of the residual distortion at the output of the receive filterbank. The optimization is carried out assuming either PR or NPR conditions as specified in Sections 5.1.1 and 5.1.2 respectively. In the next subsections, we provide a more specific characterization of the optimization procedure that we propose to follow in these two different circumstances.

### 5.3 Prototype pulse optimization under perfect reconstruction (PR) conditions

In this section, we propose to design the prototype pulse coefficients in order to minimize the residual distortion power at the output of the receive filterbank under the assumption that PR conditions hold. In this situation, it was shown in Section 5.1.1 that the
residual distortion power is proportional to the quantity $\eta(p)$ defined in (5.4). Hence, if the PR conditions hold, one can optimize the pulse coefficients so that the residual distortion power is minimized regardless of the channel frequency response. The basic idea consists in minimizing the cost function $\eta(p)$ under the constraint that PR conditions hold on the prototype $p[n]$.

Traditionally, two distinct approaches have been proposed in order to guarantee that the optimization process is constrained to PR prototypes. The most classical approach follows from formulating the problem as a classical minimization of a non-linear cost function subject to the constraints in (5.1), see [52]. It can be seen that the constraints in (5.1) are quadratic in the pulse coefficients, so that the original problem corresponds to the optimization of a non-linear cost function subject to quadratic constraints. This problem is highly non-linear and non-convex, but can be efficiently tackled using, e.g., Sequential Quadratic Programming (SQP) algorithms [56]. The basic approach consists in using a Taylor approximation of the original problem to convert it into a sequence of sub-problems, each one corresponding to an optimization of a quadratic cost function with linear constraints. Under certain regularity conditions, and if the algorithm is initialized at a point close enough to the global solution, the sequence of solutions of the approximate sub-problems will converge to the solution of the original one. However, in general terms, convergence to the global optimum is not guaranteed and, even in the situations where convergence does occur, it may be extremely slow. On the other hand, the intermediate solutions are not guaranteed to belong to the feasibility region, which means that in practice we might end up with some pulse coefficients that do not guarantee perfect reconstruction of the composite filterbank.

All these problems can be significantly mitigated by exploiting the parameterisation of paraunitary filterbanks presented in [51] and used latter in [53][57]. More specifically, let $p[n]$ be a symmetric prototype pulse of length $MK$ such that the reconstruction constraints in (5.1) hold. Assume that $P_{\ell}(z)$ and $P_{\ell,M}(z)$ are the $z$-transforms of the $\ell$th and $(\ell + \frac{M}{2})$th polyphase component of $p[n]$ respectively. Then, it can be shown that there exist $K$ angular parameters, denoted by $\theta_{\ell,1}, \ldots, \theta_{\ell,K}$, such that

$$
\begin{bmatrix}
P_{\ell}(z) & P_{\ell,M}(z)
\end{bmatrix} = \begin{bmatrix}
\cos \theta_{\ell,1} & \sin \theta_{\ell,1}
\end{bmatrix} \prod_{k=2}^{K} \begin{bmatrix}
1 & 0 \\
0 & z^{-1}
\end{bmatrix} \begin{bmatrix}
\cos \theta_{\ell,k} & \sin \theta_{\ell,k} \\
\sin \theta_{\ell,k} & -\cos \theta_{\ell,k}
\end{bmatrix}.
$$

Using this parameterisation, we can reformulate the original optimization problems so that the cost functions depend on this new set of variables, namely $\{\theta_{\ell,k}, \ell = 1, \ldots, M/2, k = 1, \ldots, K\}$. The main advantage of using this parameterisation stems from the fact that all the obtained solutions will always be feasible, in the sense that all prototype pulses obtained using this approach belong to the set of pulses for which (5.1) holds exactly.

This new parameterisation also provides a substantial reduction of the total number of parameters (degrees of freedom) that are available in order to optimize the prototype pulse. However, this amount is still far too large for scenarios with a relatively large number of subcarriers, e.g. $M = 512$ or higher. Such a large parameter space leads to high computational requirements in the optimization process and often results in convergence to
local extrema. In order to avoid this, [58] made the interesting observation that, in practice, the optimized angular coefficients \( \{ \theta_{k, \ell} \} \) turn out to resemble continuous functions of \( \ell \) for all the above discussed optimization criteria. This motivated the authors to approximate

\[
\theta_{k, \ell} = \theta_k \left( \frac{2\ell + 1}{2M} \right)
\]

where \( \theta_k (x) \) is a smooth function on the domain \([0,1]\). The fact that \( \theta_k (x) \) is a smooth function implies that it can be uniformly approximated with high precision by a polynomial of sufficiently large degree, i.e.

\[
\theta_k (x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \ldots + \alpha_N x^N
\]

(5.7)

where \( N \) is the degree of the polynomial and where the real-valued parameters \( \alpha_0, \ldots, \alpha_N \) are adjusted so as to minimize the approximation error. Now, by using this approximation we have transformed the original set of \( MK/2 \) parameters into a much reduced set of parameters \( \{ \alpha_0, \ldots, \alpha_N \} \). It must be pointed out that typically polynomials of degree \( N = 5 \) provide very satisfactory results almost identical to the originally optimized ones, even for relatively high values of the number of subcarriers. This compact representation therefore provides a substantial simplification of the pulse optimization process.

### 5.3.1 Optimization results

We carried out a numerical search of the pulse coefficients that minimize the distortion power expression in (5.4) for different values of the number of subcarriers and the overlapping factors, assuming a polynomial expansion in (5.7) with six coefficients. In all these cases, the initial point for the optimization was taken to be the Bregovic-Saramäki pulses as defined in [57]. These pulses are obtained by minimizing the out-of-band radiated power in (5.5) with respect to all the angles in (5.6), so that no polynomial approximation of the angles \( \theta_{k, \ell} \) was used. In our case, the initial values of \( \alpha_0, \ldots, \alpha_N \) were obtained by seeing the optimized angles \( \theta_{k, \ell} \) as functions of \( \ell \) and projecting these functions onto the corresponding functional space generated by the six first order monomials \( (1, x, \ldots, x^5) \) with \( x = \frac{2\ell + 1}{2M} \). Having fixed the initial values of the coefficients \( \alpha_0 \), the optimization was carried out using the Nelder-Mead simplex optimization method. This operation was repeated 20 times, each time using a slightly perturbed initial point for the optimization algorithm. In the end, the optimized pulse with less distortion power \( \eta(p) \) was selected. It turned out that all perturbed initializations led to local minima, so that the best initialization was always the one obtained from the classical Bregovic-Saramäki approach.

In Figures 5-1 and 5-2 we represent the traditional (Bregovic-Saramäki) versus the proposed optimized pulses in both the time and frequency domain in a situation where the number of subcarriers was fixed to \( M = 512 \) and the overlapping factor to \( K = 3 \) and \( K = 4 \) respectively. We observe that the proposed pulses are quite similar to the traditional ones, although they have larger secondary lobes and slightly better stopband attenuation.
Figure 5-1. Time and frequency domain representation of the traditional (Bregovic-Saramäki) pulses, together with the optimized ones. The number of subcarriers was 512 and the overlapping factor was fixed to 3.

Figure 5-2. Time and frequency domain representation of the traditional (Bregovic-Saramäki) pulses, together with the optimized ones. The number of subcarriers was 512 and the overlapping factor was fixed to 4.
In order to test the performance of these pulses in a realistic frequency selective channel environment, we simulated the corresponding FB-MC/OQAM system under multiple realizations of the Extended Vehicular A (EVA) channel model [59] in a noiseless scenario. Figures 5-3 and 5-4 represent the distribution of the signal to distortion ratio (SDR) measured at each subcarrier for different values of the system bandwidth. The plots on the left represent the cumulative distribution function of the SDR in dB, whereas the plots on the right represent the probability density function of the SDR gain achieved with the proposed pulse design. In general, we see that the achieved performance gains are very moderate, with performance improvements of up to 1-1.5 dB in the best situations. Furthermore, the significant SDR gains are concentrated on the high values of the SDR, which correspond to the situations where the distortion has less impact on the actual system performance. All this indicates that the traditional Bregovic-Saramäki pulse designs are indeed quite robust to the presence of channel frequency selectivity. Furthermore, the limited number of degrees of freedom that are left after imposing PR constraints are not sufficient to guarantee a very strong reduction of the residual distortion at the output of the analysis filterbank. We will see in the next subsection that higher improvements can be achieved if the PR conditions are relaxed.

![CDF of the SDR per subcarrier (dB)](image1.png)

![Distribution of the SDR gain (dB)](image2.png)

Figure 5-3. Performance of the traditional and proposed pulses under an EVA channel model (512 subcarriers, overlapping factor fixed to 3). On the left, the cumulative distribution function of the signal to distortion ratio (SDR) for different transmission bandwidths. On the right, probability density function of the SDR gain achieved with the proposed design.
5.4 Prototype pulse optimization under nearly perfect reconstruction (NPR) conditions

We have seen in previous section that the potential performance gain that can be obtained from the pulse optimization under PR conditions can be quite limited in practical settings. However, as mentioned above, PR conditions are not really necessary in practical communication settings because any residual distortion power below the noise floor will have no effect at all in the expected performance of the multicarrier system. For all these reasons, it seems reasonable to study possible optimization options when the reconstruction conditions are allowed to be only approximately met. As shown in Section 5.1.2, when the PR conditions are not met, the residual distortion at the output of the receive filterbank will depend on both the prototype pulse coefficients and the channel frequency response. In this section, we will study how to design the prototype pulse coefficients so that, given either a certain channel realization or a set of channel statistics, the total distortion is minimized.

In order to deal with the optimization problem under NPR conditions, we will follow the same approach as in [54] and construct the prototype pulse using the frequency sampling approach. This approach was initially established in [60] and later developed in [61][43]. The main idea behind this approach is quite simple: the impulse coefficients of the prototype pulse are obtained by taking the IFFT of a targeted frequency response, which is uniformly sampled at $MK - 1$ equispaced frequencies $\omega_k = \frac{2\pi}{2MK} k$. If $a_k$ denotes the magnitude of the desired frequency response at the $k$th frequency, and if we concentrate on real-valued symmetric FIR prototypes, one can express the time-domain pulse samples as [30]
\[
p[n] = \frac{2}{MK} \left( a_0 + 2 \sum_{k=1}^{MK-1} a_k (-1)^k \cos \left( \frac{2\pi k}{MK} n \right) \right).
\]

Note that we may have other responses depending on whether the number of sampled frequencies is lower than, equal to, or larger than the length of the prototype pulse, see [54] for further details. It was also shown in [30] that the continuity of the pulse in the time domain is guaranteed if the following condition holds:

\[
a_0 + 2 \sum_{k=1}^{MK-1} (-1)^k a_k = 0.
\]

Now, in order to guarantee that the pulse will be low pass and will present a significantly low stopband, one may typically fix

\[
a_k = 0 \quad k = K, ..., MK/2 - 1.
\]

If this is the case, one should only fix here the remaining coefficients, namely \(a_1, ..., a_k\) so that (5.9) holds and also the reconstruction conditions are approximately met. To guarantee NPR conditions, one typically chooses \(a_1, ..., a_k\) so that the prototype pulse approximately has the power-complementarity property. This basically means that the total sum of the set of pulse power spectra centred at each of the system subcarriers is equal to one. As shown in [30], when (5.9) holds, this can be expressed as

\[
a_0^2 + a_k^2 + a_{K-k}^2 = 1, \quad k = 1, ..., \left\lfloor K/2 \right\rfloor.
\]

When the overlapping factor is equal to \(K = 3\) or \(K = 4\), the above system of equations has a unique solution that is given by

\[
K = 3 \Rightarrow a_1 = 0.911438, \quad a_2 = 0.411438
\]

\[
K = 4 \Rightarrow a_1 = 0.971960, \quad a_2 = \sqrt{2}, \quad a_3 = 0.235147.
\]

When \(K > 4\) the number of free parameters is higher than the number of constraints, and several optimization techniques can be used to determine the set of coefficients \(a_1, ..., a_k\), see further [60].

A slightly different approach is taken in [43], where the continuity constraints in (5.8) are not imposed. This gives an additional degree of freedom that can be used to optimize the pulse coefficients according to several criteria, even when the overlapping factor is equal to \(K = 3\) or \(K = 4\).

### 5.4.1 Our approach

Here, we propose to follow the above pulse design procedure but we will relax the zero coefficient constraint in (5.9) in order to increase the number of degrees of freedom of the optimization process. We will always impose the continuity constraint in (5.8) and we will design the non-zero coefficients \(a_k\) so that the pulse presents nearly perfect reconstruction conditions. Furthermore, we will always leave one free parameter in the design that will be
tuned so as to minimize the residual distortion power. Let us specify the design procedure for different values of the overlapping factor.

5.4.1.1 Case $K = 2$

We will assume that the frequency domain response of the pulse is zero for sampling frequencies higher than $k > K + 1$, i.e.

$$a_k = 0, \quad k = K + 2, \ldots, MK/2 - 1.$$  

Hence, we generally have four non-zero coefficients, namely $a_0, a_1, a_2, a_3$, that need to be specified. The power-complementarity property holds at the sampling frequencies if

$$a_0^2 + 2a_2^2 = 1$$

$$2a_1^2 + 2a_3^2 = 1.$$  

This, together with the continuity condition $a_0 - 2a_1 + 2a_2 - 2a_3 = 0$ results in a system of three equations with four unknowns, which results in a single free parameter. We will consider $a_0$ as this free parameter. When this parameter is fixed, we can obtain the rest as

$$a_2 = \sqrt{\frac{1-a_0^2}{2}}, \quad a_1 = \frac{1}{2} \left( \frac{a_0 + 2a_2}{2} \right) + \frac{1}{2} \sqrt{\frac{(a_0 + 2a_2)^2}{4}}, \quad a_3 = \frac{1}{2} \left( \frac{a_0 + 2a_2}{2} \right) - \frac{1}{2} \sqrt{\frac{(a_0 + 2a_2)^2}{4}}.$$  

Now, it can be shown that this pulse specification is well designed as long as we restrict the free parameter to the interval $a_0 \in \left[0, \sqrt{\frac{1}{3}}\right]$.

5.4.1.2 Case $K = 3$

When the overlapping factor is fixed to $K = 3$, we will take

$$a_k = 0, \quad k = K + 1, \ldots, MK/2 - 1$$

so that here as well we have four non-zero coefficients, namely $a_0, a_1, a_2, a_3$, that need to be specified. The power-complementarity property is met at the frequency sampling points whenever we have

$$a_0^2 + 2a_2^2 = 1$$

$$a_1^2 + a_3^2 = 1.$$  

This, together with the condition $a_0 - 2a_1 + 2a_2 - 2a_3 = 0$ leads to a system of three equations with four unknowns, meaning that there remains a single degree of freedom, $a_0 \in [0, 1]$, and where the other coefficients are specified as

$$a_3 = \sqrt{\frac{1-a_0^2}{2}}, \quad a_1 = \frac{1}{2} \left( \frac{a_0 - 2a_1}{2} \right) \sqrt{\frac{(a_0 - 2a_1)^2}{2}}, \quad a_2 = \frac{1}{2} \left( \frac{a_0 - 2a_1}{2} \right) \sqrt{\frac{(a_0 - 2a_1)^2}{2}}.$$  

It can be seen that these coefficients are all positive.

5.4.1.3 Case $K = 4$

When the overlapping factor is fixed to $K = 4$, we take
so that here as well we have five non-zero coefficients, namely $a_0, a_1, a_2, a_3, a_4$, that need to be specified. The power-complimentary property is met at the frequency sampling points whenever we have

\[
\begin{align*}
  a_0^2 + 2a_4^2 &= 1 \\
  a_1^2 + a_3^2 &= 1 \\
  2a_2^2 &= 1
\end{align*}
\]

This, together with the continuity condition $a_0 - 2a_1 + 2a_2 - 2a_3 + 2a_4 = 0$ leads to a system of four equation with five unknowns, which results in one degree of freedom, which we take to be $a_0$. The rest of the parameters can be established as

\[
\begin{align*}
  a_2 &= \frac{1}{\sqrt{2}}, & a_4 &= \sqrt{\frac{1-a_0^2}{2}}, & a_1 &= \frac{1}{2} \left[ B + \sqrt{2 - B^2} \right], & a_3 &= \frac{1}{2} \left[ B - \sqrt{2 - B^2} \right]
\end{align*}
\]

where

\[
B = \frac{1}{2} a_0 + a_2 + a_4.
\]

One can easily show that all these definitions make sense if we restrict the free parameter to the interval $a_0 \in \left[ \frac{2\sqrt{2}}{5}, 1 \right]$.

**5.4.1.4 Case $K = 5$**

We finally consider the case where the overlapping factor is equal to $K = 5$. In this situation, we will force

\[
a_k = 0, \quad k = K, ..., MK/2 - 1
\]

and therefore we have five non-zero coefficients, namely $a_0, a_1, a_2, a_3, a_4$, that need to be specified. The power-complimentary property holds for the sample frequencies if

\[
\begin{align*}
  a_0^2 &= 1 \\
  a_1^2 + a_4^2 &= 1 \\
  a_2^2 + a_3^2 &= 1
\end{align*}
\]

This, together with the continuity condition $a_0 - 2a_1 + 2a_2 - 2a_3 + 2a_4 = 0$ leads to a system of four equation with five unknowns, which results in one degree of freedom, which we may take to be e.g. $a_4$. The other parameters are fixed as $a_0 = 1$ and

\[
\begin{align*}
  a_1 &= \sqrt{1-a_4^2}, & a_2 &= \frac{1}{2} \left[ B + \sqrt{2 - B^2} \right], & a_3 &= \frac{1}{2} \left[ B - \sqrt{2 - B^2} \right]
\end{align*}
\]

where $B = -\frac{1}{3} - a_4 + a_1$. These definitions are valid as long as $a_4 \in \left[ 0, \frac{2\sqrt{2}}{5} \right]$. 

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5.4.2 Optimization results

We simulated a FB-MC/OQAM system with 64 subcarriers and a variable overlapping factor ranging from $K = 2$ to $K = 5$. To illustrate the optimization process, we first considered two different realizations of an Extended Vehicular A (EVA) channel model, taken in two distinct situations where the system bandwidth was fixed to 1.5 MHz (Figure 5-5) and 5 MHz (Figure 5-7) respectively. In Figures 5-6 and 5-8 we represent the resulting average signal to distortion power ratio as a function of the free parameter for these two channel responses respectively. Observe that in general terms the optimum value of the adjustable parameter strongly depends on the channel frequency response. For the specific cases where the overlapping factor is equal to 3 and 4, we generally observe that the optimum parameters tends to the classical values equations (5.10)-(5.11) when the channel frequency selectivity becomes more severe. This can be clearly seen in Figure 5-8, where the minimum distortion for these two values of the overlapping factor is always attained at $a_0 = 1$, which corresponds to the choices in (5.10)-(5.11).

![Magnitude response of the channel (dB)](image1)

![Phase response of the channel (rad)](image2)

Figure 5-5. Channel response used in the pulse optimization process of Figure 5-6. It corresponds to a random realization of the EVA channel model for a system bandwidth equal to 1.5MHz.
Figure 5-6. Signal to Distortion Power Ratio as a function of the free parameter for different values of the overlapping factor. The frequency response of the channel was the one shown in Figure 5-5.

Figure 5-7. Channel response used in the pulse optimization process of Figure 5-8. It corresponds to a random realization of the EVA channel model for a system bandwidth equal to 5MHz.
In order to quantify the potential gains with respect to classical frequency sampling designs, we considered a high number of channel realizations and studied the cumulative distribution function of the resulting signal to distortion power ratio. Results are plotted for different system bandwidths in channel models drawn from the Extended Pedestrian A (EPA) and EVA profiles. Figures 5-9 and 5-10 show the results for an overlapping factor equal to $K = 3$, whereas Figures 5-11 and 5-12 were obtained for an overlapping factor equal to $K = 4$. The signal to distortion power ratio (SDR) achieved with the optimized parameter is compared to the SDR of the traditional pulses in [61], which are referred to as “Phydias pulse”. These traditional pulses can be obtained from the above designs by simply fixing $a_0 = 1$.

In general terms, we can observe that significantly higher gains in SDR can be achieved by relaxing the PR constraints and allowing NPR prototypes. Traditional designs available for the case of $K = 3$ and $K = 4$ appear to be rather optimal in situations where the channel frequency selectivity is rather severe. Finally, it is worth pointing out that the proposed optimization technique could be extended to other prototype pulse specifications different from the ones obtained with the frequency sampling technique.
Figure 5-9. Cumulative distribution function of the signal to distortion ratio in a scenario with 64 subcarriers and overlapping factor equal to 3. The channels were generated according to the EPA model.

Figure 5-10. Cumulative distribution function of the signal to distortion ratio in a scenario with 64 subcarriers and overlapping factor equal to 3. The channels were generated according to the EVA model.
Figure 5-11. Cumulative distribution function of the signal to distortion ratio in a scenario with 64 subcarriers and overlapping factor equal to 4. The channels were generated according to the EPA model.

Figure 5-12. Cumulative distribution function of the signal to distortion ratio in a scenario with 64 subcarriers and overlapping factor equal to 4. The channels were generated according to the EVA model.
6. Conclusions

In this report we presented the orthogonality conditions for non-uniform spacing and unequal subchannel bandwidths, and introduced and substantially elaborated the variable multi-mode filter bank concept, based on fast-convolution, going beyond just the basic design.

The spectral containment of different waveforms (enhanced OFDM, polyphase and FC implementations of FBMC/OQAM, and a pragmatic approach for FMT) has been evaluated. It was observed that FB-MC waveforms greatly exceed the spectral characteristics of the best considered enhanced OFDM schemes (time-domain windowing together with cancellation carriers), while the implementation complexity of the latter one is expected to approach that of the most effective FB-MC schemes. However, it is important to keep in mind that the excellent spectral characteristics of FB waveforms may be severely distorted by the inevitable RF impairments, most notable the nonlinearity of the transmitter power amplifier. This issue will be addressed in future EMPhAtiC deliverables.

The prototype filter impulse response optimization with respect to the minimization of frequency selective channel impulse response impact on degradation of sub-channel orthogonality has been covered here as well.

Comparative evaluation of the uniform filter bank structures based on the FC-FB approach and the conventional poly-phase filter banks has been conducted in terms of implementation complexity on FPGA. While the FC-FB approach has potential for lower complexity in terms of very basic complexity metrics, it was found to have very similar complexity as the polyphase structure in the FPGA implementation in the considered 1.4 MHz LTE-like scenario. Yet FC-FB exhibits greatly increased flexibility as it is able to accommodate simultaneously different waveforms with adjustable bandwidths and data rates, even spectrally efficient configurations with non-symmetric subchannel spectra.

Based on the contributions of EMPhAtiC WP2, it has been agreed that the demonstrator development in WP9 will be based on the FC-FB implementation of the FBMC/OQAM waveform. From the communication performance point of view, FC-FB can be seen as an implementation scheme for FB-MC, e.g., channel estimation, synchronization and multiantenna processing related studies can equally well be done using the basic FBMC/OQAM model. Meanwhile, investigations for reduced complexity implementations based on traditional filter bank structures will be continued, and the studies regarding the new staggered waveforms will also be considered. Also studies of the pragmatic, time-windowing based FMT model with high commonality with CP-OFDM will be continued.

One of the main targets of the next deliverable is to perform more complete comparison of the interesting waveforms, particularly in the PMR context, based on practical scenarios and spectrum masks.
7. References


[59] LTE; evolved universal terrestrial radio access (e-UTRA); base station (BS) radio transmission and reception. Technical Report TS 36.104 v11.4.0 release 11, 3GPP, 2013.


### Glossary and Definitions

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>AFB</td>
<td>Analysis Filter Bank</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BER</td>
<td>Bit Error Ratio</td>
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<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
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<tr>
<td>BRAM</td>
<td>Block Random Access Memory</td>
</tr>
<tr>
<td>CC</td>
<td>Cancellation Carrier (OFDM sidelobe suppression method)</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
</tr>
<tr>
<td>CP-OFDM</td>
<td>Orthogonal Frequency Division Multiplex with Cyclic Prefix</td>
</tr>
<tr>
<td>DC</td>
<td>Direct Current</td>
</tr>
<tr>
<td>DFT_MC</td>
<td>Discrete Fourier Transform based Multi(ple) Carrier</td>
</tr>
<tr>
<td>DFT-SC</td>
<td>DFT-based Single Carrier</td>
</tr>
<tr>
<td>EGF</td>
<td>Extended Gaussian Function</td>
</tr>
<tr>
<td>EPA</td>
<td>Extended Pedestrian A (channel model)</td>
</tr>
<tr>
<td>EVA</td>
<td>Extended Vehicular A (channel model)</td>
</tr>
<tr>
<td>EVM</td>
<td>Error Vector Magnitude</td>
</tr>
<tr>
<td>FB-MC</td>
<td>Filter Bank Multi-Carrier</td>
</tr>
<tr>
<td>FB-SC</td>
<td>Filter Bank Single Carrier</td>
</tr>
<tr>
<td>FBMC/OQAM</td>
<td>Filter Bank Multi Carrier with Offset QAM subcarrier modulation</td>
</tr>
<tr>
<td>FC</td>
<td>Fast Convolution</td>
</tr>
<tr>
<td>FC-FB</td>
<td>Fast Convolution Filter Bank</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency Division Multiple Access</td>
</tr>
<tr>
<td>FLO</td>
<td>Frequency-Limited Orthogonal</td>
</tr>
<tr>
<td>FMT</td>
<td>Filtered Multi-Tone</td>
</tr>
<tr>
<td>GFDM</td>
<td>Generalized frequency division multiplexing</td>
</tr>
<tr>
<td>(I)DFT</td>
<td>(Inverse) Discrete Fourier Transform</td>
</tr>
<tr>
<td>(I)FFT</td>
<td>(Inverse) Fast Fourier Transform</td>
</tr>
<tr>
<td>IOTA</td>
<td>Isotropic Orthogonal Transform Algorithm</td>
</tr>
<tr>
<td>I/Q</td>
<td>In-phase/Quadrature (data, signal components)</td>
</tr>
<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>MAC</td>
<td>Multiple Access Control</td>
</tr>
<tr>
<td>NC-OFDM</td>
<td>Non-Contiguous OFDM</td>
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<tr>
<td>NRP</td>
<td>Near Perfect Reconstruction</td>
</tr>
<tr>
<td>NU</td>
<td>Non-Uniform</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<tr>
<td>OFDM/OQAM</td>
<td>Orthogonal Frequency Division Multiplexing with Offset QAM</td>
</tr>
<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
</tr>
<tr>
<td>OFDP</td>
<td>Optimal Finite Duration Pulse</td>
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<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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<tr>
<td>OQAM</td>
<td>Offset Quadrature Amplitude Modulation</td>
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<tr>
<td>PAPR</td>
<td>Peak-to-Average Power Ratio</td>
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<tr>
<td>PCC</td>
<td>Polynomial Cancellation Coding</td>
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<td>PMR</td>
<td>Professional Mobile Radio</td>
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<tr>
<td>Acronym</td>
<td>Definition</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<tr>
<td>PSK</td>
<td>Phase-Shift Keying</td>
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<tr>
<td>PR</td>
<td>Perfect Reconstruction</td>
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<tr>
<td>PTS</td>
<td>Partial Transmit Sequence</td>
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<tr>
<td>QPSK</td>
<td>Quadrature Phase-Shift Keying</td>
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<tr>
<td>RAM</td>
<td>Random Access Memory</td>
</tr>
<tr>
<td>RB</td>
<td>Resource Block</td>
</tr>
<tr>
<td>RC</td>
<td>Raised Cosine</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>RRC</td>
<td>square Root Raised Cosine</td>
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<tr>
<td>SC</td>
<td>Sub-Carrier or Sub-Channel</td>
</tr>
<tr>
<td>SFB</td>
<td>Synthesis Filter Bank</td>
</tr>
<tr>
<td>SC-FDMA</td>
<td>Single Carrier Frequency Division Multiple Access</td>
</tr>
<tr>
<td>SDR</td>
<td>Signal to Distortion Ratio</td>
</tr>
<tr>
<td>SPR</td>
<td>Subchannel Power Ratio</td>
</tr>
<tr>
<td>SQR</td>
<td>Sequential Quadratic Programming</td>
</tr>
<tr>
<td>SW</td>
<td>Subcarrier Weighting (OFDM sidelobe suppression method)</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular Value Decomposition</td>
</tr>
<tr>
<td>TLO</td>
<td>Time-Limited Orthogonal</td>
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